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BENJAMIN FRANKLIN: STUDENT OF LIFE.*

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I.

So many aspects of Franklin's life, thought, and work have already been discussed by the preceding speakers in this series that there should be little left to say. But perhaps I have one advantage over the others. They have all been limited to specific subjects, whereas I am asked to talk about life—or at least, Franklin's reading of life, which might lead me into almost anything. But I shall try to limit myself to an attempt to define the point of view which seems to me to have been at the root of all his many and varied actions, and thereby provide one more comment on that sense of wholeness and unity of character which we all feel in his presence but which we all find so difficult to define. However many avenues of his thought and experience one follows out to their manifold expressions, the return trip brings one always to the same source. Franklin asked only one question of life and of the things in it: "Does it work?" The method

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of his thinking seems to me always to be pragmatic. He relies in every problem upon experience in the immediate sense as his final authority.

If this seems too simple a statement of the question and too easy an answer, my excuse is that I am not a philosopher, that I do not believe Franklin to have been one, and that I do not look to philosophy in the strict sense of the term to provide much more than some of the language of the discussion that I have undertaken. It is the habit of literary critics and historians to describe people in terms of the main-springs of their actions and thought. Such a statement as I have made about Franklin is perhaps as much a comment upon myself and my tribe as upon him. We find ourselves constantly using such phrases as "philosophy of life," "reading of life," and "romantic and classic," to the alarm and disgust of more disciplined thinkers. Because literature involves the emotions as well as the mind, the terms which are usually applied to the definitions of thinkers are inadequate. They must either be abandoned or have their meanings stretched to include emotional attitudes as well as rational systems. When I call Franklin "pragmatic," therefore, I am attempting to describe his whole personality and the meaning of his attitude toward life. In this sense I am treating Franklin as a literary figure, which undoubtedly he is, rather than primarily as a philosopher, scientist, statesman, or social critic, even though I may give much attention to these aspects of his thought and pay little or none to the form and style of his writings.

The pigeon-holing of a great man in so summary a fashion must always imply the phrase "it seems to me." If you will consent to my calling his attitude "pragmatic," I shall try to define and to illustrate what the word may mean within the limits of this discussion. If we are satisfied that the elusive sense of unity which is so obvious to most students of Franklin has been thereby defined, we may raise the further questions of whether this attitude is characteristically, even though not exclusively, American, and whether it is a satisfactory philosophy of life in itself—in short we may ask whether pragmatism works.

2.

To say that Franklin was a pragmatist is not to imply that he would even understand the accusation were he alive today. As far as I know, the term has been in use in this sense for scarcely more than half a century. In 1907, William James called it "a new name for some old ways of thinking,"[1] and proceeded to put it into the philosopher's every-day vocabulary as one more mystification for the layman. It is the same system of thought that has more recently been called instrumentalism because, in the words of James himself, in it "theories thus become *instruments*, not answers to enigmas." It is little more than a point of view, "the attitude of looking away from first things, principles, 'categories,' supposed necessities, and of looking toward last things, fruits, consequences, facts." The result is that "ideas (which themselves are but parts of our experience) become true just in so far as they help us to get into satisfactory relation with other parts of our experience."[2] Ideas, then, have no intrinsic value apart from experience; they are valid only in so far as they can be proved useful in practice. They therefore cannot form a philosophy in the usual sense.

If Pragmatism is not in the full sense a philosophy today, it was even less so in Franklin's time; and Franklin, who did not even have this term by which to describe his point of view, was not a philosopher. It was customary for his contemporaries to call him one, and he himself founded a society which he called philosophical. Recent biographers have continued to use the term in describing him, but it must be remembered that the word "philosopher" in the eighteenth century often meant "natural philosopher," or what we should mean by the term "natural philosopher." This was Franklin's own use of it when he named as resident members of the American Philosophical Society a physician, a botanist, a mathematician, a chemist, a mechanician, a geographer, and a general natural philosopher.[3] This last Franklin himself unquestionably was, and his membership in foreign scientific societies was fit tribute to his attainments in this field. But metaphysician or logician he was not.

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[1] William James delivered a series of lectures on Pragmatism at the Lowell Institute in 1906 and at Columbia University in 1907. These lectures were published in 1907 under the title: *Pragmatism: A New Name for Some Old Ways of Thinking*. Although *The New English Dictionary* gives seven earlier uses of the term, its use in philosophy was invented by C. S. Pierce in discussions with James, and its special meaning was expounded by him in an article "How to Make our Ideas Clear," *Popular Science Monthly*, January, 1878.

[2] W. James, op. cit., Lecture II, "What Pragmatism Means," pp. 53, 54–55.

The danger of attempting to explain Franklin's mind in terms of systematic philosophy is illustrated by a recent analysis of his thought by Chester Jorgenson who strives to attach him to what he calls "scientific deism," a metaphysical system based upon Newtonian physics and the rationalism of Locke. Mr. Jorgenson develops his thesis at some length and concludes: "To see the reflection of Newton and his progeny in Franklin's activities, be they economic, political, literary, or philosophical, lends a compelling unity to the several sides of his genius, heretofore seen as unrelated." His *modus operandi* is best explained "in reference to the thought pattern of scientific deism." [4] This theory is undeniably suggestive if it be used merely as an explanation of the formative period of Franklin's thought, but Mr. Jorgenson has neglected to give full weight to Franklin's emphatic rejection of the naive system-building of his youth. His short period of philosophical inquiry, 1725–29, ended with his *Modest Inquiry into the Nature and Necessity of* [not Liberty, or God, but] *a Paper Currency*.[5]

Related in so formal a sense I do not believe that the sides of his genius were. To be sure, Franklin absorbed the spirit of inquiry and experiment from the intellectual atmosphere of his day and he may justly be thought of as a product of the eighteenth century enlightenment, but to assume from this that he formulated in his own mind and held to a philosophical basis for his actions is to push a half-truth too far. It would be difficult to document the statement that "Franklin was a disinterested scientist in the sense that he interrogated nature with an eye to discovering its immutable laws."[6] As far as I know, Franklin only once attempted an intellectual and systematic statement of his philosophy. This was in his early pamphlet, *A Dissertation on Liberty and Neccessity, Pleasure and Pain* (1725), published when he was only nineteen. By following the logic of his own propositions through, he reached conclusions which seemed to him not to work. Among his surprising results are the statements that, "If there is no such Thing as Free-Will in Creatures, there can be neither Merit nor Demerit in Creatures," and that, "Pleasure is equal to Pain." "I printed a small Number," the old man wrote in looking back over the follies of his youth. "It occasion'd my being more consider'd by Mr. Palmer, as a young Man of some Ingenuity, tho' he seriously Expostulated with me upon the Principles of my Pamphlet which to him appear'd abominable. My printing of this pamphlet," he concludes, "was another Erratum."[7] That was the beginning and the end of the philosopher Franklin.

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[3] "A Proposal for Promoting Useful Knowledge among the British Plantations in America." Phila.: May 14, 1743. *Writings*, ed. by A. H. Smyth, New York: 1905–07, II, 228–232.

[4] *Benjamin Franklin, Representative Selections*, with an Introduction, Bibliography, and Notes, by Frank Luther Mott and Chester E. Jorgenson. New York: 1936. Introduction, p. cxli.

[5] *Writings*, II, 133–154.

But another kind of sage was born at that moment, one who lived rather than formulated his thought. Fifty years later he explained to Benjamin Vaughan that "the great uncertainty I found in metaphysical reasonings disgusted me, and I quitted that kind of reading and study for others more satisfactory."[8] If we wish, therefore, to discover a "compelling organic unity" in his thought, we must seek it rather in his attitude and actions than in his expression of theories. The sense of wholeness with which he impresses us is perhaps more accurately attributed by Mr. Van Doren to the fact that "his powers were from first to last in a flexible equilibrium."[9] Where should we look for the secret of that poise?

3.

The first and most obvious field for our inquiry is of course that of scientific experiment, which he insisted in carrying out in spite of the protests of the Good Mouse Amos, who lived, according to Robert Lawson, in Ben's fur cap.

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[6] Ibid., p. cxi.

[7] "Autobiography," *Writings*, I, 277–78.

[8] Passy, November 9, 1779. *Writings*, VII, 412.

[9] C. Van Doren, *Benjamin Franklin*. New York: 1928, p. 782.

"I shall tear the lightening from the skies," Amos quotes him as saying, "and harness it to do the bidding of man."

"Personally," said the mouse, "I think the sky's an excellent place for it." [10]

The familiar stories of the Pennsylvania fireplace, or the electric kite, or any of the other examples of Franklin's investigative mind would any of them reveal the process of thinking in which I am here interested. Apparently he read the reports of investigations being conducted elsewhere and kept up a correspondence with other experimental scientists, but of theoretical reading there is little record except in his youth. His correspondence with Collinson, Kames, Hartley, and other European thinkers is full of discussions of scientific experiments and political and economic developments, with some practical morality, and no metaphysics. Phenomena themselves first attracted him—smoky chimneys, the common cold, oil on water, lightning striking barns and steeples. Then followed a few simple experiments to determine the conditions under which each operated. When he had satisfied his curiosity on this point, he did not, as would most experimental scientists, formulate a law or hypothesis and push the problem further into theoretical regions. Rather, he turned about and sought to harness the lightning with rods and wires, make a ladder for his bookshelves, or work out a system of diet and exercise that would check the common cold. Such abstractions as atoms, calories, and vitamins would have had no interest for him. But he was tireless in the observation of phenomena. In a letter to Lord Kames, June 2, 1765, he observes that after wearing green spectacles the page he is reading appears reddish, but draws no conclusions except that here is a relationship between green and red "not yet explained." [11] In another letter, to the Abbe Soulevie, he expresses his preference for that "method of philosophizing, which proceeds upon actual observations, makes a collection of facts, and concludes no further than those facts will warrant." [12] He disciplined his imagination to discover only observable facts and then asked to what use his findings might be put to improve the lot of man, and more particularly of Philadelphians.

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[10] R. Lawson, *Ben and Me*. Boston: 1939, p. 41.

[11] *Writings*, IV, 380.

[12] Ibid., VIII, 601.

It would seem that Franklin's mind ran a similar course with reference to social thinking. His fire company, his postal service, his library, and his newspapers and magazines at home and in other colonies were developed to answer the public need rather than for personal profit alone. Experience in each case pointed out a situation which needed the exercise of his ingenuity, and his solution to the problem was so obvious, once it had been put into practice, that it immediately became public habit. Often it was a scientific discovery which was turned into a social channel and the two currents of his mind flowed together. He had no consistent view of the nature of society other than that dictated by his understanding of the needs of his own country. Rousseau's Sophie, the Comtesse d'Houdetot, made him the hero of a *fête champêtre* at her house at Samois, but the social reformer Franklin seems to have shown more interest in the lady than in the theories of the social idealist Rousseau. Adam Smith's *Wealth of Nations*, which appeared in 1776, does not seem to have had much direct influence, although he had met its author in 1759 at the house of Dr. Robertson in Edinburgh. When he reached the conclusion that American independence was inevitable, he joined Jefferson in approving an agricultural economy. "There seem to be but three ways for a nation to acquire wealth," he wrote. "The first is by war, as the Romans did, in plundering their conquered neighbours. This is robbery. The second is by commerce, which is generally cheating. The third is by agriculture, the only honest way, wherein man receives a real increase of the seed thrown into the ground, in a kind of continual miracle." [13] Such a simplification of Physiocratic doctrines could only be made by a man who had his eye fixed upon the practical problems of a young and unexploited country.

In the education of youth, he revealed the same lack of concern for abstract theory, the same practical and far-sighted wisdom with reference to fact. In his several tracts on educational matters, he stresses the need for the establishment of academies in the colony and outlines a pragmatic curriculum in which facility in speaking and writing, and the reading of contemporary and recent English literature, share with history and natural science the places habitually assigned to logic, theoretical mathematics, and the classics. He even urges that, with this study, excursions might be made "to the neighbouring Plantations of the best Farmers, their Methods observ'd and reason'd upon for the Improvement of Youth."[14] The modern "activity" school which has developed from John Dewey's pragmatic theories of education, with its emphasis upon the study of the immediate environment, is largely a rediscovery of practices which Franklin advocated in 1749. Higher education, with its diversified vocational schools, has followed a similar pattern. To Franklin, it was enough that America needed young men to carry forward the material welfare of the colonies and their people.

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[13] *Writings*, V, 202.

A similarly pragmatic attitude is to be discovered in his political thought. Such men as Tom Paine and Jefferson were left to absorb the theories of the French political radicals and to draft the Declaration of Independence while Franklin sat back behind his square spectacles and merely helped it to do its work by writing perhaps his most stinging and famous satire, "Rules by Which a Great Empire may be Reduced." Conciliator that he had been up to this time, he was ready to add a barb to the shaft. Apparent inconsistencies in his position during the years when the revolutionary movement in the colonies was taking shape are explained when reference is made to his fact-finding approach to the problem rather than to any systematic political philosophy. Professor Verner Crane has analyzed this problem so carefully and convincingly in the last of his Colver Lectures,[15] and doubtless again in his lecture in this series, that I shall merely agree here with his main point: that Franklin's first ideal was one of federated imperialism, but that colonial loyalty led him to alter his actions in terms of developing circumstances which were beyond his control. The really significant conclusion is not whether he held to this or that political philosophy, but that in the crisis he used his far-sighted understanding of men and events to lead rather than to follow colonial thinking into channels which brought the most satisfactory results, within the limits of possibility. He changed his plan for action with changing circumstances.

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[14] *Proposals Relating to the Education of Youth in Pennsylvania.* Phila.: 1849. Ibid., II, 386–396.

[15] V. W. Crane, *Benjamin Franklin, Englishman and American.* Providence, R. I.: 1936.

4.

When we turn from consideration of scientific, social, and political questions to the more subjective realms of ethics and human relationships, the problems become more difficult and more subject to misunderstanding. This is the real test of the theory I have proposed.

I once asked a loyal scholar of Franklin why he was so much interested in him, why he was willing to devote so much time to a study of him and of his ideas. The answer was immediate and spoken with firm conviction, "Because he knew how to deal with women." If I repeated my question here, I should probably receive a variety of answers, but I suspect that many of them would be variations on the theme, "Because he knew how to deal with life." We are attracted to many great figures of the past because of their ideas or their works; Franklin, I think, draws us because he so obviously worked out a rule of life which brought an unusual degree of satisfaction to him and to most people with whom he came into contact.

There are two kinds of sources which might be used in reformulating Franklin's attitude on personal experience and moral conduct: the record of his relationships with other people, and his own formulation of his ethical code. On a superficial view, the conclusions derived from one of these kinds of sources do not agree with those based on the other. There appears to many people an inconsistency between the experimenter with life and people, who emerges from a review of the biographical facts, and the dogmatic moralists of the *Autobiography*, *Poor Richard*, and some of the letters. So great has this inconsistency appeared to some people that poor old Ben emerges from their studies as little more than a smug hypocrite. "Although I still believe that honesty is the best policy," writes the Englishman D. H. Lawrence, "I dislike policy altogether; though it is just as well not to count your chickens before they are hatched, it's still more hateful to count them with gloating after they *are* hatched. It has taken me many years and countless smarts to get out of that barbed wire moral enclosure that Poor Richard rigged up. Here am I now in tatters and scratched to ribbons, sitting in the middle of Benjamin's America looking at the barbed wire, and the fat sheep crawling under the fence to get fat outside and the watchdogs yelling at the gate lest by chance anyone should get out by the proper exit. Oh America! Oh Benjamin! And I just utter a long loud curse against Benjamin and the American corral.

"Moral America! Most moral Benjamin. Sound, satisfied Ben!" [16]

There has never been a more wrong-headed comment made on Franklin, yet it is easy to see why this romantic, mystical Englishman, who doubtless knew no other Franklin than that he extracted from the *Autobiography* and the *Almanacks*, should rebel against what seemed to him to be a self-appointed Chief Justice of Human Nature:

"Eat not to fulness; drink not to elevation.

"Lose no time, be always employed in something useful; cut off all unnecessary action.

"Avoid extremes, forebear resenting injuries as much as you think they deserve."

Do what you want to do, but don't do too much of it—a "Do" and a "Don't" in every sentence! No wonder that Lawrence found smugness here, and tried to free himself from the inhibitions he had learned from Poor Richard!

But he completely missed the fact that these moral dogmas were not set down for him. In their first form, they were merely working guides for the young man Franklin, not a final statement of his ideal of perfection. They furnished a means of pinching himself in the arm when he found himself doing something of which he, in the long run, could not approve. And, from his own testimony, such occasions were not rare. Many a good New Year's resolution would make a bad law of the land. The most that the old man Franklin hoped was that some of these little instruments of conduct might be useful to some of his descendants as well. They had been sharpened by his own experience and were of value to him chiefly as a way of paring down his own excesses. He recorded the whole experiment with the detachment of a scientific observer, himself the student and himself the object of study. His proposed book, *The Art of Virtue*, which he never wrote, was intended, he explains, to help those who had lost the better support of Christian faith, to retain at least a working morality.[17]

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[16] D. H. Lawrence, *Studies in Classic American Literature.* New York: 1923, p. 21.

The frankness of the *Autobiography* has charmed and troubled the critics from the start; charmed because, in simple Addisonian English, Franklin set to work like an honest tradesman to take an inventory of his life, its successes and failures; troubled because the tradesman included in his count the qualities of spirit which for some have an untouchable sancity. Which is the prude: the man who can evaluate his impulses and measure their consequences or the man who looks upon his own ego as the mysterious, untouchable mystery of the eternal? The elements which make up conduct are no more exempt from Franklin's analysis than are those which control the phenomena of nature. The same scales must be used for weighing one's self that have proved their worth in the weighing of objective nature. Father Abraham's speech (*Poor Richard's Almanack* for 1758) preaches frugality and industry as practical answers to the question of high taxes.[18]

The difficulty of appraising Franklin's moral attitude lies, therefore, in this apparent inconsistency between his obviously experimental way of living and the codified system of controls which he attempts in the early pages of the *Autobiography* and which finds expression in the sayings of Poor Richard and elsewhere. In his writings, Franklin apparently states a theory of conduct first and urges others to accept and apply it to themselves; in his own actions he seems to reverse the process, to work experimentally toward an inner equilibrium of desire and control without deliberate formulation of principles until after the fact. The inconsistency is a real one if his statements on moral questions be accepted primarily as an effort to guide others. Unquestionably he is himself to blame. He played the part of the moral teacher, he gave the appearance of wishing to pass on to others his own rules of conduct. But, at the same time, this impression of Franklin as a dogmatic moralist is misleading. Before he was twenty-three he had weighed pleasure against reason in the characters of Philocles and Horatio. The resulting rule of moderation was still his fifty years later when he wrote "The Whistle." It was a rule of living which would always allow a pragmatic test, always be flexible.[19] It is, therefore, important to judge him not so much by what he seemed to be or by what he thought himself to be, as by what he was, and I think it safe to say, even in the face of all the moral aphorisms that dot his pages, that in his own life he tested conduct as he did nature by the experimental method, balancing reason and pleasure anew in each new circumstance. In this view, his proverbs, epigrams, and rules become merely the laboratory note-book of a pragmatic moralist; not texts from a secular pulpit.

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[17] Letter to Lord Kames. *Writings*, IV, 12–13.

[18] *Writings*, III, 408.

"He that falls in love with himself will have no rivals." Franklin knew that he was very much in love with himself.

"The most exquisite folly is made of wisdom spun too fine." He had tried it himself and given it up.

"We can give advice but we cannot give conduct." He had seen his own admonitions disregarded, as were Father Abraham's.

Such aphorisms as these have the appearance of chemical formulae derived from the test tube. Even the moralistic passages from the *Autobiography* may be so interpreted because Franklin is as conscientious in reporting his failures as he is his successes in living up to his own ideals and rules of conduct. But this interpretation of his moral attitude can scarcely be avoided when the facts of his conduct are reviewed apart from the evidence of the *Autobiography*. It is his lack of dogmatism which charmed in all his personal relationships, his receptivity to the ideas of others, his adjustability to their moods. His comment on his two old Junto friends,

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[19] *Writings*, II, 157–170.

Potts and Parsons, would suggest that he recognized things in human nature which wisdom could not alter: "Parsons was a wise man that often acted foolishly; Potts a wit that seldom acted wisely. If *enough* were the means to make a man happy, one had always the means of happiness without ever enjoying the thing; the other had always the thing without ever possessing the means. Parsons even in his prosperity always fretting; Potts in the midst of his poverty always laughing. It seems, then, that happiness in this life rather depends on internals than externals; and that, besides the natural effects of wisdom and virtue, vice and folly, there is such a thing as a happy or an unhappy constitution." [20] This is indeed an admission for one who supposedly believed so fully in the correctability of human nature.

Not unlike the misunderstanding of Lawrence is that of those recent critics who have labeled Franklin "the first civilized American," and "the apostle of Modern Times," but the error is of a contrary sort.[21] Phillips Russell and Bernard Fäy revolted, as did Lawrence, against the sanctimonious lay-preacher of the mis-read *Autobiography*. But instead of merely condemning him, they attempt apologies by playing up in a deplorably sensational fashion his wordliness, his cosmopolitan vices, his doubts, and his weaknesses. The resulting pictures are as distorted as that which they set out to correct. The chief value of these books, in spite of the superficiality of the one and the supposedly documented thoroughness of the other, is that they free Franklin from the clutches of Mrs. Grundy and restore him to the more congenial society of Madame Brillon. Ford [22] had allowed the latter lady's confession that she sat on the sage's lap, but other evidence even more damaging was as yet unrevealed. After the bad taste and the misrepresentation of the Russell book had opened the way, it was possible for Mr. Van Doren and others to discuss, without apology, the presence of illegitimacy in the Franklin family to the fourth generation, cut short there by untimely death. And after the Fäy book, it was easier to appreciate Franklin's life as a struggle with himself and with circumstances. The starch was washed out of the bourgeois saint.

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[20] *Writings*, III, 457.

[21] P. Russell, *Benjamin Franklin, the First Civilized American*. New York: 1926. B. Fäy, *Franklin, the Apostle of Modern Times*. Boston: 1929.

[22] P. L. Ford, *The Many-Sided Franklin*. New York: 1899.

Such revelations and interpretations are in themselves of no moment except to prove once and for all that Franklin's moral aphorisms were expressed with conviction because moderation was for him an acquired art rather than a result of prudish inhibitions. They reassure us that he was a human being who developed character and kindliness by facing life squarely and evaluating it in its various and contradictory moods as it presented itself to him. The moralizing of the old man was the summary of a life which had been richly, if not always wisely or even admirably lived. *Poor Richard* was the note-book of a laboratory moralist.

It is hardly necessary to review for this audience the many evidences of Franklin's kindly interest in others, particularly in young people, throughout his long life. Mr. Stifler's [23] collection of his correspondence with Polly Stevenson and the Shipley girls, covering more than thirty years, is almost testimony enough. But to this may be added a list of all those young men whom he stimulated and aided to successful lives through the Junto, by means of letters of introduction as in the case of Tom Paine, and by setting them up in business for themselves after an apprenticeship with him. And if evidences of his tolerance be sought, his treatment of Arthur Lee and Silas Deane, both of whom he refused openly to condemn even when the tide of popular feeling was strong against them, might be sufficient. It may be argued that good business sense and self-interest led Franklin to set his partners up in the printing and publishing business in neighboring colonies, but no such explanation can be given for his willingness to accept the risk of public stigma rather than to denounce the weaknesses of his diplomatic associates.

These instances are taken from widely spread epochs in his life, and the later ones reveal even more patience and kindliness than do the earlier ones. Franklin verified by experience his early belief that more flies are caught by molasses than by vinegar, that a world in which people get on with each other is happier and more comfortable than one in which they do not. Perhaps this point of view is sufficient explanation of the change in his political philosophy from a belief in British federation to one in American independence. His moral philosophy was as pragmatic and adjustable as was his political.

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[23] J. M. Stifler, *My Dear Girl; the Correspondence of Benjamin Franklin with Polly Stevenson, Georgiana and Catherine Shipley.* New York: 1927.

5.

In no sphere of experience, however, is this trial-and-error method of Franklin more dramatically revealed than in that of religion. Those who would accept his guidance for the whole of life must follow him through to this, and it is here that many of his warmest admirers are forced to stop. Because he was so nearly successful in depriving God himself of his mystery, he shocks many a potential disciple who admires his experimental approach to problems of science, society, and politics, and even those who are willing to apply with him the same methods to the problems of human conduct.

A careful analysis of Franklin's religious beliefs and practices would demand a review of 18th century Deism, but once more I shall be content with a discussion of his attitude alone. Again he is more interested in a workable practice of religion than in the formulated dogmatisms and skepticisms of his age. Brought up in a society which accepted a dogmatic Calvinism, the same revolt which took him to Philadelphia carried him over spiritually to the company of the religious doubters who owed their Deistic beliefs to Newton and Locke in the first instance. His first experiments in formulating a faith of his own show the traces of both influences, but they proved unsatisfying. He soon gave up the attempt to define the nature and scope of the power of God when his reasoning brought him to the conclusion that freedom of the will was a logical impossibility but a practical necessity. A system of living founded upon such premises would not work and was therefore not for him. Yet he felt the need of a God to whom he could pray. The resulting Deity was little more than a fellow-traveler, one who stood ready with his higher authority to sanction the actions which experience taught him were best. Franklin, I believe, never defined his Deity in these words, but in all his contacts with religious sects and religious people, he asked only that whatever God they might profess would prove his goodness by directing them into the wise and virtuous ways of living. He tells us that, religiously educated a Presbyterian, he attended no services for public worship with any regularity because he became impatient with theological discussion. Nevertheless he recognized the practical virtues of the churches as agencies for the public welfare, and was willing to contribute to any sect that would serve the civil interests as well as their own. For the same reason he welcomed and supported George Whitefield when he came to Philadelphia in 1739, and his justification for joining the Masons was that God judges men more for what they do than for what they think.[24] His *Articles of Belief and Acts of Religion* (1728),[25] written at the age of 22, posits a Diestic Christian God who has infinite power but is above using it arbitrarily, and a curious hierarchy of beings superior to man that is suggestive of pagan pantheism. The most important clause in this creed, however, is the statement that God is good and wise, and therefore Benjamin Franklin of Philadelphia sees every reason for making Him his friend. In this spirit of independence, he proceeds to his devotions as he would to a conversation with a superior moral adviser for whom he feels deep reverence. All of his later religious efforts, including his proposed abridgement of the *Book of Common Prayer*, were designed to make more easy and immediate the every-day intercourse between man, the superior of the animals, and God, the supreme in wisdom and judgment of all beings. But in his most religious moods, this intercourse was still in the nature of a conversation between one being and another. Surely no one has ever devised a more helpful God, but the experience of religious exultation is lacking in the relationship. It was for immediate and practical aid that he urged the Constitutional Convention of 1786 to turn to God, and the psychological effect of his suggestion was good.[26] Franklin's pragmatic attitude seems to stand this final test; he created and clung to a God who helped him discover how to live, and allowed him to make the discovery for himself. With other problems of the nature and function of the Deity he early lost interest. And when Ezra Stiles asked him a month before his death, to state his opinion on the Divinity of Christ, he replied: "It is a question I do not dogmatize upon, having never studied it, and think it needless to busy myself with it now, when I expect soon an Opportunity of knowing the Truth with less Trouble."

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[24] Letter to Josiah Franklin, April 13, 1838. *Writings*, II, 214–16.

[25] *Writings*, II, 91–100.

[26] M. Farrand, *The Records of the Federal Convention of 1787*. Rev. ed. 4 v. New Haven: 1937, I, 450–52.

6.

Franklin's way of thinking and living may be distinguished from those of his European contemporaries like Voltaire and Johnson in that it was so characteristically American. The particular form which his pragmatism took was a result of his participation as a principal actor in the greatest mass movement of civilization that history records. Lewis Mumford summarizes it in a sentence: "The settlement of America had its origins in the unsettlement of Europe." [27] Never before had man made so deliberate and so ambitious an effort to transfer a matured civilization to a primitive land. The movements of Greek culture to Rome and of Latin culture to England are trifling when compared to the gigantic task of transferring the culture of Western Europe to American shores. Nor was the contrast between the sophistication of the culture and the barbarism of the land ever so striking. Historians have labeled the effort "the frontier movement" and have more and more, since Frederick Jackson Turner defined it in 1893, come to realize that much in American cultural history can be better explained when referred to this basic factor. Franklin lived at a time when the frontier culture of the eastern seaboard colonies was reaching maturity through a painful adolescence. As a result, his pragmatism has a peculiarly American flavor, a vitality and aggressiveness which the tired skepticism of contemporary European thinkers lacked. Whatever his predilections may be toward a life of contemplation and dogmatic belief, the American philosopher from the earliest days has been forced out of his assumptions and into a mold of vigorous pragmatism by the very circumstances of his life. However sound his philosophy when kept in the realm of theory, and however admirable his love of security and retirement for contemplation, the immediate need for action has always been too great to allow any system of thinking which cannot justify itself by providing swiftly the needs and the minimal comforts of life. "America was promises," sings our latest Laureate Archibald MacLeish; but the American is pragmatist. He must make some of the promises work.

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[27] L. Mumford, *The Golden Day*. New York: 1926, p. 11.

I should like to ask your indulgence for perhaps too long a digression at this point because I feel that we can only appreciate Franklin's pragmatism by discovering the same trait in other Americans and seeing it as a dominant strain throughout the long history of our intellectual development.

It takes little argument to prove that the original settlers in this country were primarily concerned with practical problems. Those who were not, did not live to produce books or descendants. Even when they were motivated by religious ideals as were the Plymouth colonists and the Pennsylvania Quakers, their first acts were such common sense translations of their theories into workable practices as the *Plymouth Compact* and the classic treaty with the Indians. And even when the colonies were fairly settled, there was more debate on problems of economics and government than upon the abstractions of philosophy and the amenities of literature. In the famous *Bloudy Tenent* controvery itself, Roger Williams was protesting persecution for cause of conscience as a working principle of throttling free speech in a democracy rather than as a scholastic dogma. It was more than a century before the colonies produced in Jonathan Edwards a philosopher who is worthy to take rank among original and systematic thinkers, and almost two hundred years before that type of literature which is concerned with the pleasures and amenities of life took shape in Cooper, Irving, and Bryant. The great theorist of the Revolution, Tom Paine, had few original ideas and devoted his energies to applying the principles which he had learned in his reading to the problems which his adopted country faced in the fact.

This unusual phenomenon of a whole nation working in accord with a philosophy which it had not, in almost two centuries, had time to formulate is commented on some years later by the astute Frenchman, de Toqueville: "I think that in no country in the civilized world is less attention paid to philosophy than in the United States," he writes. "The Americans have no philosophical school of their own; and they care but little for all the schools into which Europe is divided, the very names of which are scarcely known to them. Yet it is easy to perceive that almost all the inhabitants of the United States conduct their understanding in the same manner, and govern it by the same rules. . . . Each American appeals to the individual exercise of his own understanding alone. . . . As it is on their own testimony that they are accustomed to rely, they like to discern the object which engages their attention with extreme clearness; they therefore strip off as much as possible all that covers it, they rid themselves of whatever separates them from it, they remove whatever conceals it from sight, in order to view it more closely and in the broad light of day."[28]

With this shrewd explanation of the American habit of pragmatic independence of mind, even such idealists as Emerson and Thoreau consent to fall into the pattern. Kenneth Murdock has pointed out that the Puritans themselves were men of affairs and tied their idealism closely into their daily lives.[29] By 1840, Puritanism had stripped itself of all the inhibitions and regimentations of theological dogma, and life could be defined and lived as a new testing of experience each day by each individual. Emerson came very near to formulating an ethical philosophy, even though he failed to systematize his metaphysics and æsthetics. In *The American Scholar*, in 1837, he preached an idealization of the moral code of which Franklin was, in many respects, an example. Impatience with books as books is countered by a plea for closer communion with nature and for action. Franklin's nature was of the human variety, but we can easily imagine him, granted Emerson's gift of tongues, speaking the following lines: "If it were only for a vocabulary, the scholar would be covetous of action. Life is our dictionary. Years are well spent in country labors; in towns; in the insight into trades and manufactures; in frank intercourse with many men and women; in science; in art; to the one end of mastering in all their facts a language by which to illustrate and embody our perceptions. . . . Life lies behind us as the quarry from which we get tiles and copestones for the masonry of to-day."[30]

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[28] A. de Toqueville, *Democracy in America*, trans. by H. Reeve. New York: 1898, II, 1–7.

[29] K. Murdock, "The Puritan Tradition," *The Reinterpretation of American Literature*, edited by Norman Foerster, New York: 1928, p. 105.

I am fully aware that in calling Emerson a pragmatist, at least in some phases of his thought, I am flying in the face of much of the traditional interpretation of his attitude, although I am not alone among his recent critics in committing this crime. Frederick Ives Carpenter writes: "Emerson's philosophy may perhaps be described as a Pragmatic Mysticism. It is idealistic in that it puts the mystical experience first. It is dualistic in that it looks both ways from its position on the bridge between soul and nature. It is monistic in that it maintains that this bridge is the only reality. But it is pragmatic in that it tests all truths (including the mystical belief in the value of life) by experience.

"It remains to suggest," continues Mr. Carpenter, "that this pragmatic mysticism is essentially *the* American philosophy." [31] With this conclusion I am inclined heartily to agree although it has been pointed out convincingly that Emerson, in his more mystical moods, rejects the experimental attitude toward physical facts which is characteristic of the pragmatism of Franklin and of other more practical people from William Penn to William James. But the discovery of pragmatism in any sense in Emerson is akin to the sensational. In him, and perhaps also in Thoreau, Whitman, and the other leaders of the mid-nineteenth century renaissance of idealism in literature and life, the mystical is more important than the circumstantial experience. But if we compare these idealists with European literary men and philosophers of the same general stamp, like Coleridge, Goethe, and Kant, the horns of the Yankee tradesman appear above the all-seeing eye.

Throughout the American experience, therefore, our chief problem has been the adjustment of an extreme form of pragmatic individualism, demanded by the circumstances of our civilization, to the fundamental hunger of human nature for something higher and better than it can obtain in this world. The purer forms of idealism have seldom taken root in our soil because of the urgency of the pragmatic challenge, especially in the early days. Franklin, more successfully than any other Colonial, faced this issue and was temperamentally able to make the adjustment which the circumstances of his time and country demanded. He reduced religion to practical ethics and he tested conduct by experience. Emerson, a mystic by temperament, founded his ethics, as his Puritan forefathers had done before him, on the same pragmatic base. It was left for William James to return to the foundations which Franklin had laid by the pattern of his life and to formulate a theory which Franklin had lived without formulation. There is much reason to believe that this modern pragmatism is the characteristic American philosophy, the one which our experience has dictated from the start. We may not each of us accept it without qualifications for ourselves, but as a people we have learned it and applied it throughout our history. We may, like Emerson and many another American, feel the need for accepting the mystical experience and explaining the universe in terms of a polarity rather than of a single point of reference. But in Franklin's singleness lay his strength and his unity. It might prove a sanitive to these troubled times and to the many troubled minds living in them to return more frequently to the study of a man whose philosophy was himself, and who discovered how to make himself whole by rejecting no part of the life about him, and sane by keeping it in control. On such a foundation we may build according to our various temperaments and needs.

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[30] R. W. Emerson, *Works* (Cent. Ed.), I, 97–98.

[31] F. I. Carpenter, *Ralph Waldo Emerson, Representative Selections*, New York: 1934. Introduction, p. xxxvii.

Two Stratospheres in Middle Latitudes.—(*Heating and Ventilating*, Vol. 39, No. 2.) The atmosphere has an attic. Over the temperate regions of the earth is a "pseudo-stratosphere," concludes DR. HENRYK ARCTOWSKE, Polish meteorologist, now conducting investigations at the Smithsonian Institution, from a study of radio sounding records of the U. S. Weather Bureau. A roof of the world long has been recognized. For a certain distance, depending on latitude, temperature decreases with altitude. With greater heights the decrease stops and it may even become a little warmer. The point where this occurs is known as the tropopause. All the atmosphere above it is the stratosphere, vaguely explored field of the air voyagers of tomorrow and, it generally has been supposed, region of almost perfect calm. The tropopause, as it long has been known, is highest over the equator. There, at any season, is the coldest spot "on earth." It slopes down, like the roof of a house, to the two poles. Four years ago the French meteorologist Jaumotte fixed the base level of the tropopause at approximately 18 kilometers above the equatorial regions. This level corresponds, according to Jaumotte, to the level of increase of helium, of ozone, and of the electrical conductivity of the atmosphere—a real, atmospheric roof of the planet. But the radio soundings show, Dr. Arctowske insists, that in middle latitudes one must admit the existence of two stratospheres. For certain distances the temperature goes down according to schedule, the rate of decrease increasing with altitude. Then there is a narrow region with little or no decrease, after which the decrease sets in again. The middle region might be compared to the world's attic.

R. H. O.

SOME REFINEMENTS IN METHODS OF GRAPHICAL INTEGRATION.

BY

LLOYD HAMILTON DONNELL, Ph.D.*

The methods of graphical integration to be described are applicable to ordinary differential equations of the first order, though they may sometimes be applied to equations of higher order, in the same manner as other similar methods. Many differential equations of the first order are impossible or impractical to integrate directly, and so a number of methods have been evolved for doing this numerically or graphically.

Numerical methods are very satisfactory in cases where the value of the derivative does not vary over too wide a range, that is when the plotted function is not very far from a straight line. In other cases numerical methods are likely to prove troublesome, and since we frequently cannot tell beforehand what our equation is going to give us, it is often something of a gamble to start with them. The graphical methods to be described, on the other hand, can be depended on to work satisfactorily in all cases. Like all graphical methods they are subject to the disadvantage that they require some skill, and are subject to drafting errors. However many people prefer graphical methods because they feel that they can see just what they are doing, whereas numerical and analytical methods must be carried out more or less blindly.

As in most methods of graphical integration we start by plotting "equal slope lines." The equations of these lines are obtained by substituting tan $n\theta$ for the derivative in the differential equation. θ is some convenient angle, such as 15°, and n is an integer having values 0, 1, 2, 3 etc. for the different equal slope lines. The quantity tan $n\theta$ is thus a constant for each line (see Table 1) and the equation obtained by substituting it for the derivative is an ordinary algebraic equation, which can usually be plotted very easily. In exceptional cases such equations can always be plotted by using well known methods of cross plotting and interpolation.

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As an example, for the differential equation

$$\left(\frac{dy}{dx}\right)^4 - y\frac{dy}{dx} + 3x^2 = 0, \qquad (1)$$

the equations of the equal slope lines are obtained by substituting values of tan $n\theta$ for dy/dx. Using $\theta = 15°$ and Table 1,

TABLE 1.

$n\theta$.	tan $n\theta$.
0°, 180°	0
15°, 195°	.2679
30°, 210°	.5774
45°, 225°	1.000
60°, 240°	1.732
75°, 255°	3.732
90°, 270°	$\infty = 1/0$
105°, 285°	−3.732
120°, 300°	−1.732
135°, 315°	−1.000
150°, 330°	− .5774
165°, 345°	− .2679

these equations are:

$$\begin{aligned} &x = 0 \quad (n = 0) \\ &y = .019 + 11.2x^2 \quad (n = 1) \\ &y = .193 + 5.2x^2 \quad (n = 2), \text{ etc.} \end{aligned} \qquad (2)$$

Each of the equal slope lines is the locus of points at which the integral curve has the *slope* tan $n\theta$, or the *angle* θ. Thus in the above example, any integral curve, that is any graph of the differential equation (1), has a zero angle with the x-axis where it crosses the first of lines (2), an angle of 15° with the x-axis where it crosses the second of lines (2), and so on.

Having drawn the equal slope lines, we are ready for the integration. This proceeds by jumps. Starting from a known point, given say by the boundary conditions of the physical problem, we jump to the next equal slope lines; but in between we must resort to guesswork, something that "looks reasonable," or to some approximation such as the methods to be described. The deviation of our line from its true position due to this uncertainty of what to do between the equal slope lines may be small at first, but the error is likely to be cumulative as we go along.

An obvious way to reduce this error is to use more equal slope lines, that is, to use a smaller value for θ. However it does not pay to carry this too far, as the more steps there are in the integration the greater are the unavoidable drafting errors; also the labor involved is more or less proportional to the number of steps. Evidently there is a happy mean somewhere and for most cases a value of 15° for θ will be found satisfactory. With this value all the angles involved can easily be obtained by combining the standard 30° and 45° draftsman's triangles.

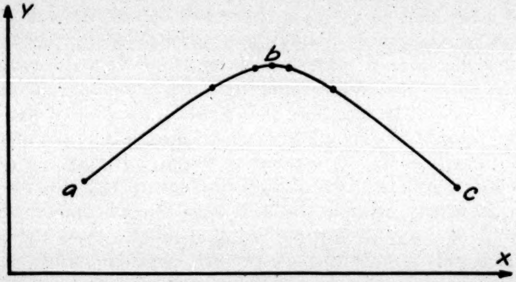


FIG. 1.

It may be noted that where the integral curve has only a slight curvature the intervals between intersections with equal slope lines will be comparatively long, whereas when the curvature is great the intervals will be short. In Fig. 1, for example, suppose that the curve *abc* is the integral curve. The dots represent the points where the curve will intersect the equal slope lines. At *a* the curve makes an angle of 45° with the x-axis, at the next dot it has an angle of 30°, etc. As the angular changes from dot to dot are equal, the distances between dots—which represent the jumps by which integration would proceed—are great where the curvature is small, near *a*, and are small where there is a sharp curvature, near *b*. This is obviously a very advantageous arrangement as it makes the steps of the integration fine where they need to be fine, and coarse where fineness is unnecessary. The usual method of constructing equal slope lines is to plot them from equations obtained by substituting more or less arbitrarily chosen constants for the derivative. This in general results in a wide variation in the fineness of the steps of the integration, with no relationship to the fineness which is desirable at each point. With the method described above, on the other hand, an even degree of accuracy at various parts of the curve is automatically assured.

If θ is taken as 15° then there will be only twelve values for tan $n\theta$, as given in Table 1, and hence no more than twelve equal slope lines to plot (unless multiple valued functions, such as anti-trigonometric functions, are involved). In many cases there will be less than twelve lines, as some of the lines will be found to be imaginary, which means that the integral curve never has the corresponding slopes. If very few of the expressions give rise to real lines, this means that the integral curve is nearly straight; in this case one of the numerical methods of integration may be found more satisfactory. In any case only a small section of each equal slope line will be actually used, and some of them may not be used at all, depending on how far it is desired to carry the integration. Hence we can often save much labor if we make a preliminary survey, by going through the whole process in a very rough manner, to get an idea what sections of the equal slope lines need to be drawn in accurately. Or we may draw in the necessary sections of the equal slope lines one at a time, as the integration proceeds and as we need them.

If the equal slope lines happen to be straight lines, it is of course comparatively easy to plot them. Sometimes a transformation of variables can be made in the differential equation which will bring about this result.

Having considered the construction of equal slope lines, let us turn to the drawing of the integral curve, satisfying the differential equation. Two methods are proposed for doing this. The first is very simple and fast and is capable of a high degree of accuracy, but its accuracy depends much on the skill with which it is used. It consists merely in finding a curve, say a section of a French curve, which will cross

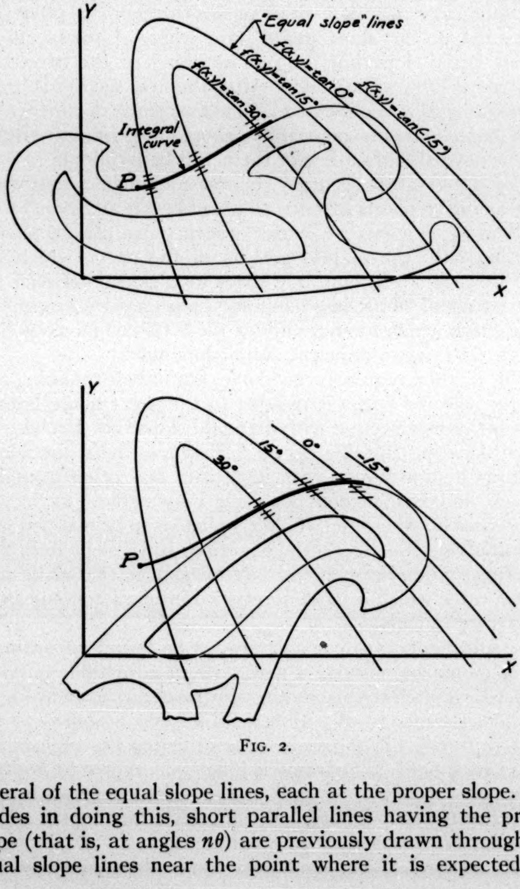

Y
"Equal slope" lines
f(x,y)=tan 0°
f(x,y)=tan 15°
f(x,y)=tan 30°
f(x,y)=tan(-15°)
Integral curve
P
X
Y
30°
15°
0°
-15°
P
X

FIG. 2.

several of the equal slope lines, each at the proper slope. As guides in doing this, short parallel lines having the proper slope (that is, at angles $n\theta$) are previously drawn through the equal slope lines near the point where it is expected the integral curve will cross. Starting at a known point the integral curve can thus be drawn in steps, quickly and with good accuracy. Figure 2 shows two steps in this process. *P* is the starting point, given say by the boundary conditions. By trial a curve is found which passes through this point, and is parallel to the short guide lines where it intersects the nearest equal slope line, and at as many of the succeeding equal slope lines as possible. After the first section is drawn in, the integral curve is extended another short distance in the same manner, and so on. If the short guide lines are drawn only as needed, only a few of them will be required.

For the greatest accuracy, curves should be found which fit at as many points at once as possible—in any case at not less than three points. Greater accuracy can also be secured by using only the middle portion of the curve which fits. Thus we may fill in only one space at a time—between two adjacent equal slope lines—but for each step we attempt to find a curve which fits not only at these two equal slope lines but also at the two adjacent equal slope lines.

This method requires a good eye, not only for making sure that the selected curve is parallel to the short guide lines at the point of intersection with an equal slope line, but also for quickly locating suitable curves. However, it is possible to obtain as high accuracy with it as with any other graphical method, and with experience it is very rapid. It is particularly useful where the whole x,y plane is to be mapped with lines satisfying the differential equation; equal slope lines with their short guide lines should be drawn over the whole area desired, after which a large number of integral curves can be drawn very quickly.

As suitable curves can not always be found on ordinary French or marine curves, a new type of adjustable curve is proposed, adapted to this or other purposes, as shown in Fig. 3. This consists merely of a thin flexible strip attached at the ends to a frame, with provisions for adjusting the angle of the strip at each end. The frame is hinged, as shown at *h* in the figure, so as to allow the ends of the strip to approach each other when the strip is bent. By suitably adjusting the angles of the ends of the strip a very wide range of curves can be secured, as suggested in the figure, and in the photographs of a model. An adjustment can also be provided at the hinge, as shown, to allow longitudinal loads to be applied to the strip, which somewhat increases the range of curves which can be secured. For small scale work the strip may be of spring brass about 3 in. long, .005 in. thick and .5 in. high. In use,

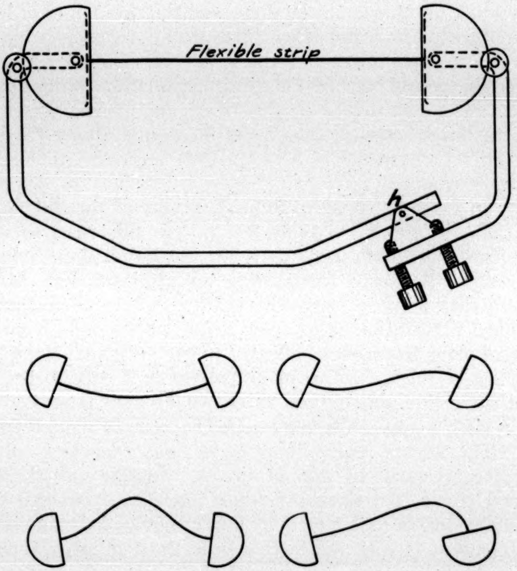


FIG. 3.

the strip should be pressed tightly against the paper with the fingers of the left hand when the pencil is applied, to prevent its moving. Or the strip may be beveled to a sharp edge at the bottom, and the impression made in the paper used instead of a pencil mark; if the paper is backed up with blotting paper a legible impression is easily secured.

The second method for drawing the integral curve is more automatic, requiring more labor but less skill than the method just described. It is a refinement of the well known method of drawing straight lines of proper slope through the equal slope lines, intersecting about half way between them, and then smoothing out the resulting jagged line. It consists in finding out *just where* the straight lines *should* intersect, instead of merely letting them intersect "about half way between." If this could be done exactly, the points of intersection with the equal slope lines would be exactly on the correct integral curve, and the result of smoothing out the jagged line would be a very close approximation to this correct curve. The method to be described, for finding where the straight lines should intersect, depends on studying the variation in the curvature of a rough first approximation to the integral curve.

As an example consider the simple case of the differential equation $dy/dx = x$. Taking $\theta = 30°$ in this case, so as to magnify the errors in usual methods, the equal slope lines are the vertical lines $x = \tan 0°$, $x = \tan 30°$, etc., Fig. 4*a*. If our starting point is, say, the origin, the usual method of graphical integration is to draw a line at 0° angle from the 0° equal slope line about half way to the 30° equal slope line; from here a line is drawn at 30° about half way to the 60° equal slope line, and so on, as shown in Fig. 4*b*. If we do this in such a way as to make lines such as *bc* and *cd* equal, then what we are really doing is to replace portions of the true integral curve by arcs of circles. In this case the true integral curve is a parabola whose curvature changes continuously, and in replacing it by arcs of circles we make errors which lead us farther and farther from the true integral curve as the work proceeds, as shown in Fig. 4*c*. If, instead of making lines such as *bc* and *cd* equal, we adopt some other arbitrary rule such as to make the intersection *c* come half way between the equal slope lines, we are in general as likely to make matters worse as to make them better. (In this particular case it happens to make them better.)

Figure 4*d* shows the true integral curve with tangents drawn at the intersections with the equal slope lines. These tangents are evidently the lines we should have drawn in carrying out our graphical integration. The lengths of lines such as *BC* and *CD* are not equal, but have a certain definite

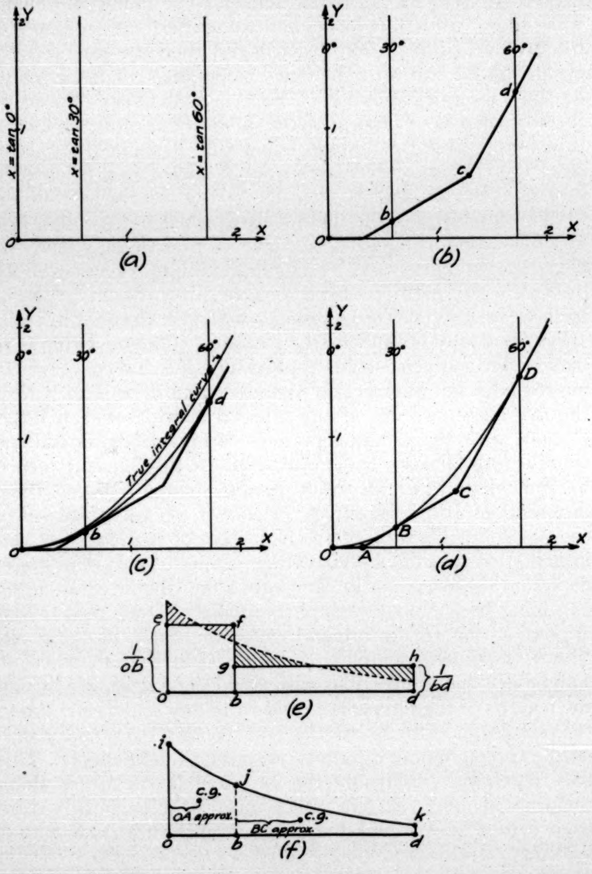

Y
X
O
1
2
x = tan 0°
x = tan 30°
x = tan 60°
(a)
0°
30°
60°
b
c
d
(b)
True integral curve
(c)
A
B
C
D
(d)
e
f
g
h
o
b
d
1/ob
1/bd
(e)
i
j
k
c.g.
OA approx.
BC approx.
(f)

FIG. 4.

ratio to each other. If we had some way of telling beforehand what these ratios should be, and we carried out each step of the graphical integration accordingly, then the intersections of our jagged line with the equal slope lines would be points on the true integral curve. The ratios obviously depend on the variation in curvature of the integral curve; if the curvature were uniform BC would equal CD; if, as in the present case, the curvature decreases from B to D, then BC will be shorter than CD, and vice versa.

The first step in the method is to construct a rough first approximation to the integral curve, so that we may study its variation in curvature. This approximate curve may be drawn by any method—even a curve drawn free-hand, so as to have roughly the proper slopes where it crosses the equal slope lines, may be sufficiently accurate. The next step is to make a development of this approximate integral curve, showing the points of intersection with the equal slope lines. Now the average curvature of the integral curve, between intersections with two equal slope lines, is inversely proportional to its length between these intersections; this is easily seen if we remember that the total angular change between these intersections is a constant, θ. Hence if, on each interval of the development, we plot the reciprocal of the length of this interval, we obtain a "curvature graph," that is a graph depicting the variation in curvature along the integral curve.

Going back to our example, suppose we took as our first approximation to the integral curve the curve *obd*, Fig. 4*c*. The development of this is the line *obd* in Fig. 4*e*, where *o*, *b* and *d* are the intersections with equal slope lines. Measuring the lengths of the intervals *ob* and *bd* to any scale, calculating the reciprocals of these lengths, and plotting these reciprocals on the corresponding intervals, we obtain the line *efgh*. This line represents graphically the variation in curvature along the original curve *obd*, Fig. 4*c*, because the average curvature from *o* to *d* is θ/ob and the average curvature from *b* to *d* is θ/bd.

The curvature of the *true* integral curve will have nearly the same average value between equal slope lines as our first approximation, but it will vary continuously instead of in jumps. A close estimate of the variation in curvature of the true integral curve is therefore obtained by smoothing up the graph *efgh*, as indicated by the dotted line in Fig. 4*e* or the line *ijk* in Fig. 4*f*. In doing this we should try to keep the *areas* for each interval the same as before, by making the similarly shaded areas in the figure approximately equal.

Having obtained the "curvature graph" *ijk*, showing to some scale (the scale does not matter) the variation of curvature along the integral curve, the desired ratio between the lengths of lines such as *BC* and *CD* is very simply obtained. It is equal to the ratio between the distances from the center of gravity of the area *bjkd* to the ends *jb* and *kd*; or in other words, the c.g. of the area *bjkd* divides the interval *bd* in the desired ratio of *BC* to *CD*. In practice the total lengths *bd* and *BCD* are usually nearly the same, especially when a small value of θ has been used. In such a case it is sufficiently accurate to take the distances of the c.g.'s from the left hand ends of the corresponding intervals as equal to the lengths *OA*, *BC* etc., as indicated in Fig. 4*f*. Hence as soon as we have determined the curvature graph, such as *ijk*, the construction of the second approximation to the integral curve is easy; we merely lay off at a 0° angle the length *OA* as determined from the curvature graph; from *A* we draw *ABC* at 30°, taking the length *BC* from the same graph, and so on.

If there is considerable difference between lengths such as *bd* and *BCD*, then we must carry out each step of the construction in such a way as to make the ratio of lines such as *BC* to *CD* equal to the ratio in which *bd* is divided by the c.g. of the area above it. This can be done with sufficient accuracy by marking the dimensions from the curvature graph on the edge of a sheet of paper and comparing them with tentative lines on the other diagram, until, by eye, the same ratio is obtained.

The determination of the c.g.'s of areas such as *oijb* and *bjkd* can most easily be carried out by changing the areas into equivalent trapezoids, as shown in Fig. 5, being careful to make the positive and negative shaded areas roughly balanced about the center, so that the location of the center of gravity will not be seriously affected. The center of gravity of the trapezoid may then be determined by well known formulas or graphical methods, as indicated in the figure.

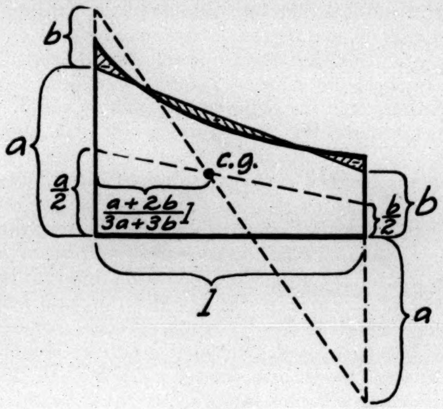

b
a
c.g.
$\frac{a}{2}$
$\frac{a+2b}{3a+3b}l$
$\frac{b}{2}$
b
l
a

FIG. 5.

The proof that the c.g. of areas such as *bjkd* divides the interval *bd* in the same ratio as *BC* to *CD* is most easily given by using an analogy with the well known Area Moments

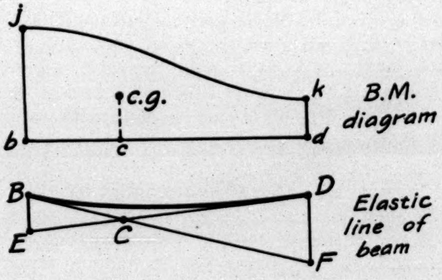

j
c.g.
k
B.M. diagram
b
c
d
B
D
Elastic line of beam
E
C
F

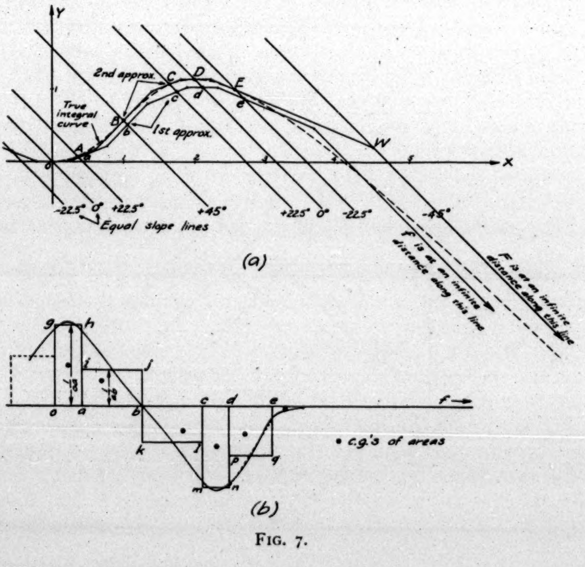
FIG. 6.

method of studying the deflection of beams. If, in Fig. 6, the curve *BD* represents the elastic line of a bent beam, with tangents *BCF* and *DCE* drawn at its ends, and *bjkd* is its bending moment diagram, having the c.g. shown and the area A, then by the Area Moment laws:

$$BE = (A/EI)bc, \qquad DF = (A/EI)cd. \tag{3}$$

Hence $bc/cd = BE/DF = BC/CD$ by similar triangles. The relation between an integral curve and its "curvature graph" is exactly the same as exists between a bent beam and its bending moment diagram, as the curvature of a beam is everywhere proportional to the bending moment. Hence the above proof applies to the similarly lettered lines in Figs. 4*d* and 4*f*. The proof is, of course, strict only if the angle between BC and CD is small, but the error is slight when this angle is 15° or even 30°.

A less crude example of the application of this method is given in Figs. 7 and 8. Again θ is taken rather large to magnify the chance for error. The figures deal with the same problem in the same way, the difference being that in Fig. 7



Y
X
2nd approx.
C
D
E
True integral curve
B
A
a
b
c
d
e
1st approx.
W
o
1
2
3
4
5
-22.5°
0°
+22.5°
+45°
+22.5°
0°
-22.5°
-45°
Equal slope lines
F is at an infinite distance along this line
f is at an infinite distance along this line
(a)
g
h
i
j
o
a
b
c
d
e
f →
k
l
m
n
p
q
• c.g.'s of areas
(b)

FIG. 7.

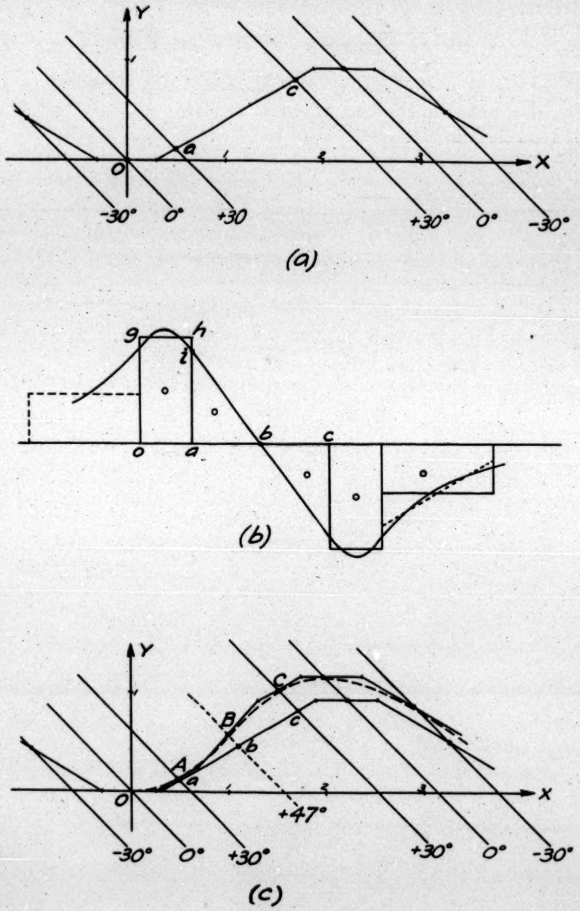

Y
X
O
1
2
3
a
c
-30°
0°
+30
+30°
0°
-30°
(a)
g
h
i
o
a
b
c
(b)
Y
X
O
1
2
3
A
B
C
a
b
c
+47°
-30°
0°
+30°
+30°
0°
-30°
(c)

FIG. 8.

θ is taken as 22.5° and in Fig. 8 as 30°. The differential equation is $dy/dx = \sin(x + y)$, with the origin as the starting point. This equation can also be integrated mathematically, and of course in practice there is seldom any point to using graphical integration in such a case. However it is better for an illustration, as we have an exact solution, shown in broken lines in the figures, with which to compare our method. This problem is a difficult test of any method, as the true integral curve has a point of inflection and approaches one of the equal slope lines asymptotically.

The rough first approximation is obtained by the method of drawing lines of proper slope through the equal slope lines and intersecting half way between them. This was chosen in order to compare the inaccuracies involved in this method with the refinement possible with the second approximation, and also because it is probably the simplest and most nearly "automatic" of all methods of graphical integration.

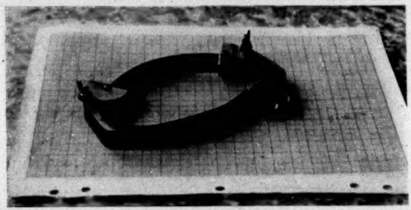
In Fig. 7*a* the points *a*, *b*, *c*, *d*, *e*, and *f* are the intersections with the equal slope lines obtained by the first approximation. The development of this first approximation is given in Fig. 7*b*. It makes little difference whether a development is made of the jagged line or of a smooth curve drawn through the points *a*, *b*, *c*, etc. It is of course much easier to develop the jagged line, as was done in this case; the points were simply marked in order along the edge of a sheet of paper held successively against each portion of the jagged line in Fig. 7*a*, and then transferred to Fig. 7*b*; this is much easier than using dividers.

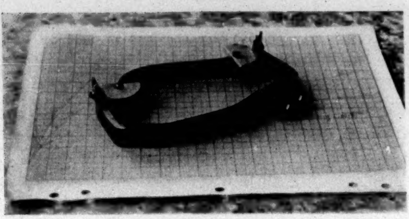
Next, the lengths of the intervals *oa*, *ab*, etc., Fig. 7*b*, were measured according to a convenient scale, their reciprocals were calculated and plotted on the corresponding intervals, giving us the jagged "curvature graph" *g h i j k l*, etc. The reciprocals are plotted in the positive or negative directions according as the curvature in the corresponding section of the integral curve, Fig. 7*a*, is concave upward or downward. There need be no confusion about this, as it makes no difference which convention is used for positive and negative curvatures, and it is always perfectly obvious when the curvature changes sign.

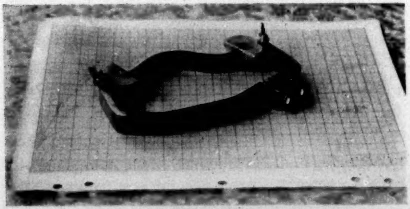
The jagged curvature graph was then smoothed up as shown, care being taken to keep the areas over each interval roughly the same. The center of gravity of the area over each interval was figured by the formula given in Fig. 6 and the second approximation to the integral curve was drawn accordingly, as previously described, giving us the points A, B, C, D, E and F in Fig. 7*a*. It will be noted that according to the construction, since the length ef is found to be infinite, the ratio of EW to WF is given as zero; hence the point F, the intersection of the integral curve with the $-45°$ equal slope line, is an infinite distance from W along this line, as it should be, since the exact curve approaches the line asymptotically.

Figure 8, in which θ is taken as 30°, illustrates a case where the method as so far described apparently fails. However if the meaning of the various steps in the process is kept in mind, then it is always quite evident in such cases what must be done. In Fig. 7 it happened that one of the equal slope lines passed through the point of inflection of the integral curve and this saved us from trouble. In Fig. 8*a* we are not so lucky. The equal slope lines at the ends of the interval ac are both $+ 30°$. Since the total angular change of the integral curve over this interval is zero, its average curvature must be zero. Hence in constructing the curvature graph, Fig. 8*b*, the ordinate for the interval ac should be zero, instead of $1/ac$ as ordinarily. There is no particular danger of making a mistake and plotting the reciprocal in such a case, because it must be decided whether to plot the ordinate in the positive or negative direction, and as soon as this question comes up it at once becomes evident that the curvature is neither positive nor negative, and hence the ordinate must be zero.

Another difficulty presents itself when we attempt to find the center of gravity of the area on ac, Fig. 8*b*. As the area is composed of equal positive and negative parts its center of gravity is an infinite distance away, and the construction of the second integral curve at this point becomes indeterminate. The obvious way around this difficulty is to insert another equal slope line at the point of inflection, as shown by the dotted line in Fig. 8*c*. The point where the curvature graph crosses the development of the first integral curve, b in Fig. 8*b*,







is plotted back onto the first integral curve, and the new equal slope line is drawn through this point, parallel or approximately parallel to the neighboring equal slope lines. The angle corresponding to this new equal slope line can readily be found from the curvature graph. The *area* under this graph between two points represents the *angular* change in the integral curve between these points. Hence we can find the angle corresponding to the new equal slope line from the area of the curvature graph between it and a neighboring equal slope line. In this case it is found by trial that the area *aib*, Fig. 8*b*, is about 17/30 of the area of any of the rectangles in the figure, each of which represents an angular change of 30°. The area *aib* therefore represents an angular change of about 17°, and the angle corresponding to the new equal slope line throubh *b* is given as 47° (17° added to the angle at *a*). Having determined this new equal slope line, the work can continue as before.

If, in Fig. 7, the points determined for the second approximation to the integral curve, *A*, *B*, *C* etc., are compared with the exact integral curve shown in broken lines, it will be seen that the errors in this second approximation are very small, in spite of the fact that θ was taken as large as 22.5° and the work was done with no more than ordinary care. The points do not lie exactly on the true line, of course, but the errors were of the order of magnitude of the width of the pencil lines used in the construction, and were therefore probably due as much to drafting errors as to errors inherent in the method. A third approximation can of course be made by constructing a development of the second approximation and so on, but experience indicates that generally this is unnecessary, and is useless unless the work is done with extreme care. The labor of carrying out this method is not very great—in fact it is much easier to do the work than to describe it. No more than ordinary drafting skill is required, and good results can be obtained even if the work on the curvature graph is done very carelessly or approximately—that is, better results are obtained by using the method roughly than by using less exact methods carefully.

OPTICAL ASPECTS OF THREE-DIMENSIONAL PHOTOELASTICITY.*

BY

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1. TWO-DIMENSIONAL PHOTOELASTICITY.

In two-dimensional photoelasticity a plate of transparent isotropic material is stressed by a uniplanar force system coplanar with the middle plane of the plate (Fig. 1). The material of the plate thereby becomes optically doubly refracting and certain of the resulting optical properties are determined by examining their effects on polarized light passed through the plate at normal incidence. The state of stress in the plate is then calculated on the basis of experimentally determined relations between stress and optical effect carried out under similar conditions.

The state of stress, at a point in a plate loaded only by forces in its plane, is a particular case of a more general stress condition; the most general being characterized by six independent quantities, namely, the magnitudes and the directions of the three mutually perpendicular principal stresses. The three principal stresses will be denoted in this paper by the symbols σ_3, σ_2, σ_1, and their relative magnitudes will be taken as $\sigma_3 > \sigma_2 > \sigma_1$.

In a plate loaded in its plane, one of the principal stresses is always normal to the plane of the plate and its magnitude is zero, so that three of the six quantities are known *a priori*. The remaining three quantities are the magnitudes of the two principal stresses, lying in the plane of the plate, and their orientation. An important characteristic of such a stress distribution is that, at each point in the plate, one principal plane of stress is parallel to the middle plane of the plate. It should be noticed that the two principal stresses in the plane of the plate may be σ_1 and σ_2 or σ_2 and σ_3 or σ_3 and σ_1.

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* From a paper presented at the Eighth Semi-Annual Meeting of the Eastern Photoelasticity Conference, December 10, 1938, Columbia University, New York City.

When polarized light is passed normally through the plate, the relative phase difference of the two waves, into which the light is physically resolved, is found to be directly proportional to the difference between the two principal stresses in the plane of the plate. Since two of the principal stresses vanish on a free boundary of the plate, the relative phase difference there is directly a measure of the third principal stress which is, of course, parallel to the edge of the plate.

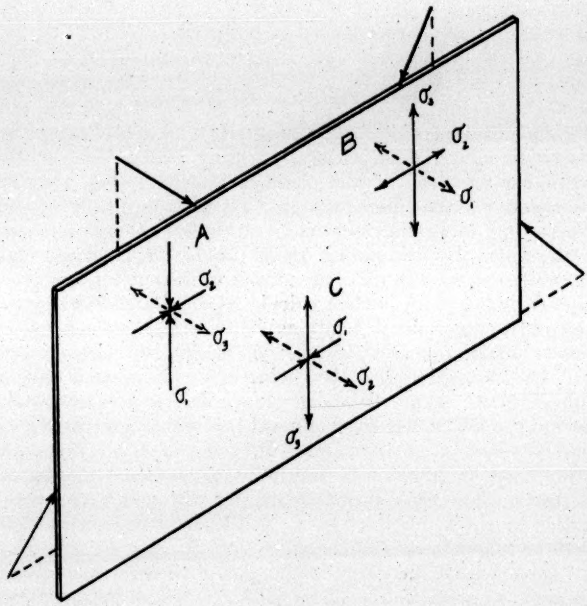

FIG. 1.

The maximum shearing stress in the plate is a quantity of prime engineering importance. It is given by half the difference between the algebraically largest and smallest principal stresses. If, at any point in the plate, the two principal stresses in the plane of the plate are of opposite sign (one tension and the other compression as at point *C* in Fig. 1), the relative phase difference is there a direct measure of the maximum shearing stress. In general, however, it cannot be said that the relative phase difference measures the maximum shearing stress in the plate. It measures the maximum shearing stress on planes normal to the plane of the plate and this will be the true maximum only when $\sigma_2 = 0$.

Experiments also show that the polarizing axes of the two-dimensionally stressed plate coincide with the axes of principal stress in the plane of the plate. Hence it is a simple matter to determine the directions of the principal stresses by observing extinction positions as the plate is rotated in its plane between crossed nicols.

Regarding the independent determination of the magnitudes of the principal stresses in a two-dimensional stress system, there is an extensive literature, to which reference is not essential for the purposes of this paper.

2. THREE-DIMENSIONAL PHOTOELASTICITY.

There is now in the course of development a tecnhique for optically analyzing three-dimensional stress systems.[1,2,3,4,5] In brief, the process consists in heating a model made from

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[1] "A New Photoelastic Material," by A. G. Solakian. *Mechanical Engineering*, **57**, 767–771 (1935).

[2] "Polarisationsoptische Untersuchung raümlicher Spannungs- und Dehnungszustände," by G. Oppel. *Forschung auf dem Gebiete des Ingenieurwesens*, **7**, No. 5, 240–248 (1936).

[3] "Polarisationsoptische Untersuchung des räumlichen Spannungszustandes im konvergenten Licht," by R. Hiltscher. *Forschung auf dem Gebiete des Ingenieurwesens*, **9**, No. 2, 91–108 (1938).

[4] "Das Kunstharz Phenolformaldeheyd in der Spannungsoptik," by A. Kuske. *Forschung auf dem Gebiete des Ingenieurwesens*, **9**, No. 3, 139–149 (1938).

[5] "Photoelastic Studies of Three-Dimensional Stress Problems," by M. Hetényi, presented at the Fifth International Congress for Applied Mechanics, Cambridge, Mass., September, 1938; also "The Fundamentals of Three-Dimensional Photoelasticity," by M. Hetényi. *Journal of Applied Mechanics*, **5**, No. 4, A-149–155 (1938).

a hardening resin (for example, a phenolformaldehyde resin or a glycerine phthallic anhydride resin) above a certain critical temperature, applying loads, cooling under load, and then removing the loads. The resulting material is optically doubly refracting and its optical properties are closely connected with the elastic state of stress present in the material when it is in the loaded condition at the elevated temperature. One of the most striking features of the process is that the optical properties of the material are apparently unchanged by cutting the specimen into thin slices. The photoelastic properties of hardening resins at elevated temperatures have been well understood as a result of the work of Hetényi (see footnote No. 5) and reference to his papers should be made for the details of the thermal process.

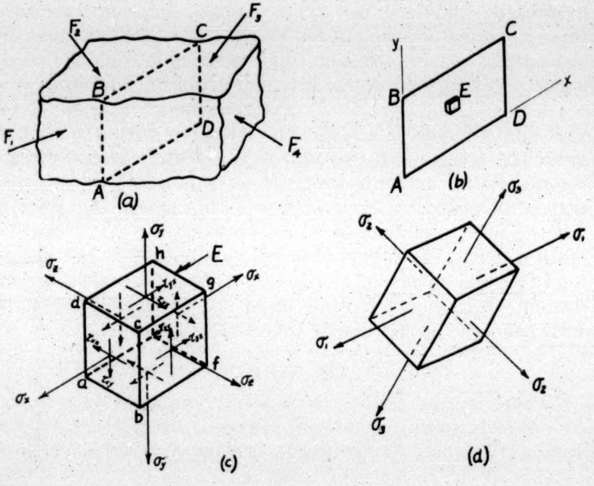

FIG. 2.

It is therefore possible to determine the stress distribution in an entire specimen by determining the optical properties of the slices. The most important characteristic of the slices, which distinguishes them from the plates employed in two-dimensional work, is that the optical properties in general represent a stress distribution involving both normal and shearing stresses on the plane of the slice. Fig. 2(*a*) represents a three-dimensional model, acted upon by a general system of forces F_1, $F_2 \cdots$, from which a slice *ABCD* is cut after the model has been carried through the thermal and loading process perfected by Hetényi. Fig. 2(*b*) represents the slice which is taken as the x, y-plane and shows a small element, E, in the shape of a rectangular parallelepiped whose faces are parallel or perpendicular to the plane of the slice. The state of stress on the element is depicted in Fig. 2(*c*). Plane *bcgf*, which is parallel to the plane of the slice, is not a principal plane of stress, since it has shearing components τ_{zx} and τ_{zy} acting on it. To obtain the orientation of the element so that only normal (principal) stresses act on it, it would be necessary to rotate it until it reaches a position such as is indicated in Fig. 2(*d*).

The relative retardation for light passed normally through the slice in the usual manner (the z-direction in Fig. 2(*c*)) is a measure of the difference between the *secondary principal stresses*, p and q, in the x, y-plane where

$$p, q = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}.$$

Since p and q are restricted to lie in the x, y-plane, they do not, in general, correspond to σ_3 and σ_2 or σ_2 and σ_1 or σ_3 and σ_1.

Hence, if there are shearing stresses on the plane of the slice, the latter is not a plane of principal stress and the relative phase difference, measured in the usual manner, is proportional, not to a difference between principal stresses but to the difference between the *secondary principal stresses* lying in the plane of the slice. The relative phase difference for the slice will be related to the principal stress difference through certain angles which fix the orientation of the slice with respect to the principal axes.

Auxiliary measurements (that is, measurements supplementing those made in two-dimensional photoelasticity) must therefore be made to obtain the data requisite for converting the phase difference of the slice to maximum phase difference. The principles involved in this process are well known to students of crystal optics and there are several methods widely used by petrologists for making the measurements. It is only necessary for workers in photoelasticity to adapt one of the petrological techniques to the requirements of photoelasticity and to ascertain the relations between the observed effects and stress.

3. SOME PRINCIPLES OF CRYSTAL OPTICS.

The optical phenomena encountered in photoelasticity are adequately described by a wave theory of light. Of the wave theories which have been devised, the electromagnetic theory is, in many respects, the most complete. Chronologically, the electromagnetic theory displaced an elastic ether theory which, before it was abandoned, had been developed to such a stage that it accurately explained all of the phenomena of crystal optics which are now observed in photoelasticity. Workers in photoelasticity are mostly civil and mechanical engineers who are, on the whole, more familiar with the stress equations of motion and the fundamental ideas of stress and strain than with the principles of electricity and magnetism. Since the former are the bases of the elastic ether theories, it seems likely that engineers will more easily master the principles of crystal optics via an elastic ether theory [6] than via the electromagnetic theory.

Regardless of which theory is used, the conclusion is reached that light is propagated in a transparent, non-isotropic medium only in the form of plane polarized waves and that, for each wave normal, there are in general two wave velocities corresponding to two waves whose polarization directions are perpendicular to each other and to the wave normal.

The wave velocities and directions of polarization may be conveniently described in terms of the geometry of *Fresnel's ellipsoid.* The semi-axes of a central section of the ellipsoid, perpendicular to a wave normal, give the reciprocals of the velocities for that wave normal; and the direction of a semi-axis of the section is the direction of polarization of the wave having the velocity given by the reciprocal of the other

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[6] See, for example, Lamé, "Théorie de l'Élasticité," Paris, 1866. Chapters XVII to XXIV.

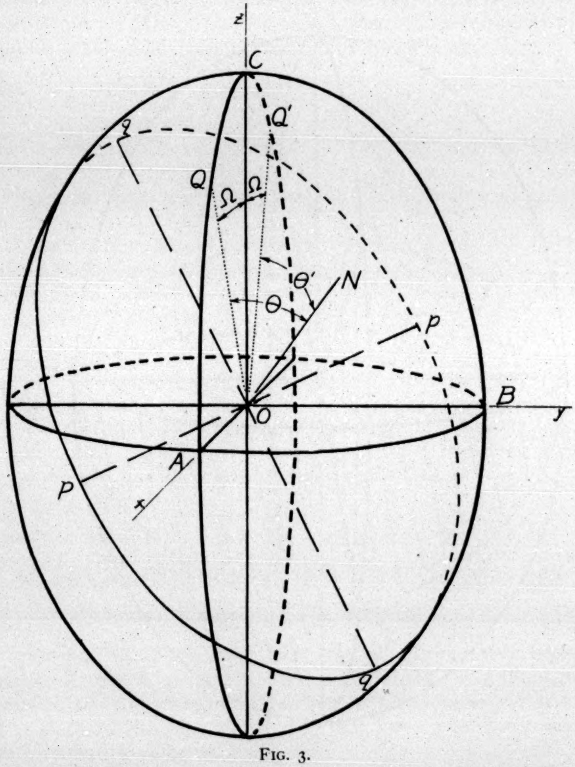

FIG. 3.

semi-axis. The three principal semi-axes of the ellipsoid, as distinguished from the two semi-axes of an arbitrary central section, thus represent the stationary values (maximum, minimum and minimax) of wave velocity. These three axes are termed the principal axes of optical symmetry of the medium.

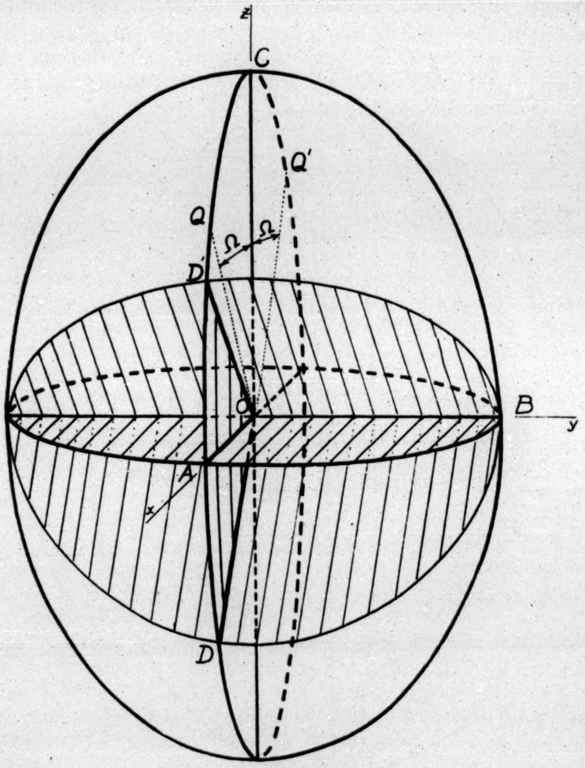

FIG. 4.

In Fig. 3, OA, OB, OC are the principal semi-axes of the ellipsoid. If a, b, and c are the magnitudes of the principal wave-velocities, then

$$OA = 1/a, \quad OB = 1/b, \quad OC = 1/c.$$

The ellipse *pqpq* is the intersection with the ellipsoid of a plane passing through the center O, the normal to the plane being ON. The seim-axes Op and Oq of the ellipse are the reciprocals of the wave velocities k_1 and k_2, respectively, for a wave normal ON. Op is the polarization direction of k_2, and Oq is the polarization direction of k_1. We also have

$$a \geqq k_1 \geqq b \geqq k_2 \geqq c.$$

There are two planes (DOB and $D'OB$ of Fig. 4) through the intermediate principal axis (OB) of the ellipsoid which intersect the ellipsoid in circles. Since the semi-axes of each of these sections are equal, a wave front parallel to either one has only one velocity. The normals (OQ and OQ' in Fig. 4) to these planes are called the optic axes; they lie in the x, z-plane and make equal angles Ω with the z-axis. The angle 2Ω is termed the *optic axial angle*. From the known geometrical properties of the ellipsoid we have

$$\sin \Omega = \sqrt{\frac{a^2 - b^2}{a^2 - c^2}}, \qquad \cos \Omega = \sqrt{\frac{b^2 - c^2}{a^2 - c^2}}. \tag{1}$$

Furthermore, if θ (Fig. 3) is the angle between the wave normal ON and the optic axis OQ, and if θ' is the angle between the wave normal and the optic axis OQ', we have, from the geometry of the ellipsoid

$$\left.\begin{aligned} k_1^2 &= \tfrac{1}{2}[a^2 + c^2 + (a^2 - c^2)\cos(\theta - \theta')], \\ k_2^2 &= \tfrac{1}{2}[a^2 + c^2 + (a^2 - c^2)\cos(\theta + \theta')], \end{aligned}\right\} \tag{2}$$

so that

$$k_1^2 - k_2^2 = (a^2 - c^2)\sin\theta \sin\theta'. \tag{3}$$

In Fig. 3, plane *pqpq* is a central section of Fresnel's ellipsoid drawn parallel to the wave front. For light traversing the slice at normal incidence, this plane is parallel to the plane of the slice, and its normal, ON, is both the wave normal and the normal to the slice.

The planes of polarization are NOp and NOq. These planes bisect, internally and externally, the dihedral angles between the planes containing the wave normal ON and the optic axes OQ and OQ'. That is, planes NOp and NOq bisect the angles between planes NOQ and NOQ'.

The most general crystal is *biaxial*, that is, there are two optic axes; but, if $b = c$ or $b = a$, the crystal becomes *uniaxial*. Considering Fresnel's ellipsoid (Fig. 3), with its greatest axis $2/c$, mean axis $2/b$ and least axis $2/a$, if we decrease the mean axis until it equals the least axis, the ellipsoid degenerates to a prolate spheroid for which the z-axis is both the axis of revolution and the optic axis. The optic axial angle (2Ω) decreases to zero so that θ and θ' are equal. This is the ellipsoid for a *positive uniaxial* crystal.

To obtain a *negative uniaxial* crystal, increase the mean axis of Fresnel's ellipsoid until it equals the greatest axis $2/c$. The ellipsoid degenerates to an oblate spheroid with the x-axis as both the axis of revolution and the optic axis. The optic axial angle increases to π and $\theta + \theta' = \pi$.

The optic axial angle, 2Ω, is defined as that angle between the optic axes which is bisected by the axis of least velocity (z). When $2\Omega < \pi/2$, the biaxial crystal resembles, somewhat, a positive uniaxial crystal and the more acute the angle, the closer is the similarity. Such a crystal is called *positive biaxial*. When $2\Omega > \pi/2$, the crystal resembles a negative uniaxial crystal and, the closer 2Ω approaches π, the closer is the similarity. Such a crystal is *negative biaxial*. When $2\Omega = \pi/2$, we have, from Eq. (1)

$$a^2 - b^2 = b^2 - c^2. \tag{4}$$

The relative phase difference, δ, of the two waves, traveling a distance ρ through the crystalline medium with velocities k_1 and k_2 is given by

$$\delta = \frac{2\pi\rho}{\lambda}(n_2 - n_1), \tag{5}$$

where n_1 and n_2 are the indices of refraction corresponding to k_1 and k_2, respectively; i.e., if k is the velocity of light *in vacuo*,

$$n_1 = \frac{k}{k_1}; \qquad n_2 = \frac{k}{k_2}. \tag{6}$$

Since the velocity differences are small in comparison with the velocities themselves, we may write, for Equations

(1), (3) and (4)

$$\sin\Omega = \sqrt{\frac{n_a - n_b}{n_a - n_c}}, \qquad \cos\Omega = \sqrt{\frac{n_b - n_c}{n_a - n_c}}, \tag{7}$$

$$n_2 - n_1 = (n_c - n_a)\sin\theta\sin\theta', \tag{8}$$

$$n_a - n_b = n_b - n_c \quad \text{when} \quad 2\Omega = \pi/2. \tag{9}$$

Hence

$$\delta = \frac{2\pi\rho}{\lambda}(n_c - n_a)\sin\theta\sin\theta'. \tag{10}$$

4. STRESS-OPTICAL RELATIONS.

If the scale of Fresnel's ellipsoid is adjusted so that a unit of length represents the velocity of light *in vacuo*, the semi-axes of any section represent indices of refraction. Experiments indicate that the same type of relation exists between stress and change of index of refraction as between stress and strain.[7] Hence, the optical properties which an originally isotropic material assumes on the application of stress may be visualized by imagining that Fresnel's ellipsoid is the result of the deformation of a sphere of radius n_0 subjected to principal stresses applied parallel to the principal axes of the ellipsoid. The mathematical statement of this principle is

$$\left.\begin{aligned} n_a - n_0 &= C_1\sigma_1 + C_2(\sigma_2 + \sigma_3),\\ n_b - n_0 &= C_1\sigma_2 + C_2(\sigma_3 + \sigma_1),\\ n_c - n_0 &= C_1\sigma_3 + C_2(\sigma_1 + \sigma_2),\end{aligned}\right\} \tag{11}$$

in which C_1 and C_2 are constants, $n_c > n_b > n_a$ and $\sigma_3 > \sigma_2 > \sigma_1$. The axes a, b, c, and 1, 2, 3 and x, y, z are coincident in the order named, i.e., a, 1 and x are coincident, etc. If C_1 is positive, a unidirectional tension ($\sigma_2 = \sigma_1 = 0$) will elongate the sphere to a prolate spheroid and so produce a positive uniaxial crystal. If, combined with this tension, a compression is applied at right angles, a positive biaxial crystal is produced when the absolute magnitude of the compression is less than the absolute magnitude of the tension. When the magnitudes are reversed, a negative biaxial crystal results.

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[7] "On the Equilibrium of Elastic Solids," by J. C. Maxwell. *Transactions of the Royal Society of Edinburgh*, **20**, 87–120 (1853).

When we have a simple shear, i.e., when absolute magnitude of the tension and compression are equal, we find, substituting $\sigma_3 = -\sigma_1$ and $\sigma_2 = 0$ in Eq. (11) and using Eq. (9), that the optic axes intersect at right angles.

It is apparent that a knowledge of the three principal indices of refraction is necessary for the calculation of the three principal stresses from Equations (11). The differences between refractive indices are generally easier to measure than the indices themselves. These differences are proportional to the principal stress differences. Thus from Equations (11)

$$\left.\begin{aligned} n_c - n_a &= C(\sigma_3 - \sigma_1), \\ n_a - n_b &= C(\sigma_1 - \sigma_2), \\ n_b - n_c &= C(\sigma_2 - \sigma_3), \end{aligned}\right\} \tag{12}$$

where $C = C_1 - C_2$.

From Equations (12), (10) and (7),

$$\left.\begin{aligned} \sigma_3 - \sigma_1 &= \frac{\lambda\delta}{2\pi\rho C} \cdot \frac{1}{\sin\theta \sin\theta'}, \\ \sigma_2 - \sigma_1 &= \frac{\lambda\delta}{2\pi\rho C} \cdot \frac{\sin^2\Omega}{\sin\theta \sin\theta'}, \\ \sigma_3 - \sigma_2 &= \frac{\lambda\delta}{2\pi\rho C} \cdot \frac{\cos^2\Omega}{\sin\theta \sin\theta'}. \end{aligned}\right\} \tag{13}$$

Of the quantities on the right in Equations (13), δ is a phase retardation which may be measured with a compensator. The wave length, λ, length, ρ, and stress optical coefficient C are determined in the usual manner. Hence, if we can measure the angles θ and θ' between the wave normal and the optic axes, we can determine the maximum shearing stress $\frac{1}{2}(\sigma_3 - \sigma_1)$. If, in addition, we can measure the optic axial angle 2Ω, we can determine all three principal stress differences.

5. SPECIAL CASES.

There are certain special cases for which it is not necessary to measure all of the three angles θ, θ' and Ω; and others for which the measurement of these angles will yield more information than simply the differences between principal stresses.

Most important is the case when the stress distribution has a plane of symmetry. This will occur when both the model and the loading system are geometrically symmetrical about a plane. The plane of symmetry is a principal plane of stress so that a slice which contains this plane will have a relative retardation proportional to one of the three principal stress differences. The plane which contains the optic axes will be either parallel or normal to the plane of symmetry. In the latter case, the bisector of the optic axial angle is either parallel or perpendicular to the plane of the slice. Hence we have $\theta = \theta' = \pi/2$ or $\theta = \theta' = \Omega$ or $\theta = \theta' = \pi/2 - \Omega$, depending on which principal plane of optical symmetry coincides with the plane of the slice. Hence, if we measure θ and Ω in addition to δ, we may calculate the three principal stress differences from Equations (13).

The two-dimensional stress system ordinarily obtained in two-dimensional photoelasticity is a special form of the case of symmetry about a plane; for, in a plate in a state of plane stress, the plane of the plate is a principal plane of stress and, in addition, the principal stress normal to the plate has zero magnitude. As in the more general case of symmetry about a plane, we measure θ and Ω (θ and θ' being equal) but now, since one of the principal stresses is zero, the other two may be determined separately from Equations (13). This method for the independent determination of the two principal stresses in two-dimensional photoelasticity was first described by Hiltscher.[3]

Hiltscher[3] also observed that, since one of the principal stresses is zero on a free boundary in a three-dimensional stress field, the other two principal stresses may be determined if θ and Ω are known.

In a prismatic or cylindrical bar subjected to St. Venant torsion,[8] one of the principal stresses at any point in the bar is zero and the other two are equal in magnitude and opposite in sign. The zero principal stress direction is perpendicular to a generator of the bar and the other two principal stress directions make angles of 45° with the generator. This state of stress corresponds to a biaxial crystal with an optic axial angle of $\pi/2$, the optic axes being parallel and perpendicular to the generator. A slice of the bar normal to a generator therefore gives zero phase difference for light at normal incidence; but a slice at any other angle to a generator will give a phase difference from which we may calculate the principal stresses. To effect this calculation, it is necessary to to know the angles (θ and θ') which the wave normal makes with the optic axes. One of these will be constant and equal to the angle between the generator and the normal to the slice, but the other will vary over the section. For each point in the slice, we need therefore measure only the latter angle (θ') and the phase difference δ. Then Equations (13) give the principal stresses directly, provided the wave-length λ, the slice thickness ρ, and the stress optical coefficient C are known. It was observed by Hetényi [5] that a 45° slice exhibits the maximum number of fringes, but it should be noted that the fringes do not represent loci of constant principal stress difference. It is necessary to know the angle θ' for each point in the 45° slice. This angle may be determined in the following manner: Let O be a point in the 45° slice, at which θ' is to be found. The normal to the slice at O and the line through O parallel to a generator of the bar determine a plane which intersects the plane of the slice in a line OA. Let OB be a polarizing axis of the slice at O (determined from the isoclinics in the usual manner) and let angle $AOB = \alpha$. Then it may be shown that

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[8] "Theory of Elasticity," by S. Timoshenko. McGraw-Hill Book Co., New York, 1934. P. 228.

$$\sin \theta' = (1 + \cos^2 2\alpha)^{1/2}.$$

Similar formulas hold for slices cut at other angles or viewed obliquely.

6. OPTICAL TECHNIQUES.

There are three standard methods for measuring n_a n_b and n_c but none of them appears to be suitable for the current development in three-dimensional photoelasticity.

In Stokes' method,[9] a plane parallel plate of the crystal is placed over a rectangular grid of ruled lines and the latter are observed through a microscope. On account of the double refraction in the plate, each family of parallel lines in the grid will appear in focus at two positions of the microscope objective. The three indices can be calculated from the relative focus positions. However, the photoelastic double refraction is so weak that this method does not appear feasible.

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[9] "Analytical Theory of Light," by J. Walker, Cambridge University Press, 1904. Pp. 225–230.

The prism method [10] would require the cutting of a prism from the model for each point at which the stress is desired.

The total reflection methods [11] are very accurate but they measure the optical properties of a surface layer of the specimen. In a photoelastic slice the surface properties are not a reliable indication of the internal optical characteristics.

For determining the differences between principal refractive indices in a doubly refracting plate, we have seen that it is necessary to measure the relative phase retardation of the plate, the optic axial angle (2Ω) and the angles (θ and θ') between the optic axes and the plate normal. The measurement of the three angles is ordinarily effected by one of two methods; one involving the use of the universal rotating stage [12] and the other involving observations with converging polarized light.[13] The latter is the method used by Hiltscher.[3] On account of the very weak double refraction of photoelastic materials, and the necessity of using thin slices, the converging light method is likely to be inaccurate if not impossible in many cases. A combination of the universal rotating stage for measuring the three angles θ, θ' and Ω with a wedge compensator for measuring δ seems to offer good possibilities for the basis of an optical technique for three-dimensional photoelasticity.

Note added in proof:

Since this paper was written, another photoelastic method for analyzing three-dimensional systems has been developed by R. Weller. Reports of recent work in this field may be found in the following papers:

R. Weller, *Journal of Applied Physics*, Vol. 10, p. 266 (1939).

R. Weller and J. K. Bussey, *Tech. Note No. 737, Nat. Adv. Comm. for Aeronautics* (1939).

R. Weller and B. Fried, *Bull. No. 106, Ohio State Univ. Eng. Exp. Sta.* (1940).

D. C. Drucker and R. D. Mindlin, *Journal of Applied Physics*, Vol. 11, p. 724 (1940).

P. R. Rosenberg, *Proc. 13th Semi-annual Eastern Photoelasticity Conference*, Mass. Inst. of Technology, Cambridge, Mass., June, 1941.

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[10] J. Walker, op. cit., pp. 230–242.

[11] J. Walker, op. cit., pp. 230–242.

[12] "Manual of Petrographic Methods," by A. Johannsen. McGraw-Hill Book Co., New York, 1918. Pp. 487–507.

[13] A. Johannsen, op. cit., pp. 449–486. See also reference No. 3.

Novel Dew-Point Recorder Safeguards Natural Gas Line Against Freezing.—(*Refrigerating Engineering*, Vol. 43, No. 3.) To To prevent condensation of moisture and possible freezing of its gas lines during the winter months, the Colorado Interstate Gas Company, which furnishes Denver with natural gas from Texas fields has built a novel dew-point recorder utilizing a General Electric photoelectric relay. With the recorder continuous readings of temperature at which condensation occurs are made where the gas enters the Denver metering plant. If calculations based on the readings show the dew-point to be too high, instructions are dispatched to dehydrating plants to remove more moisture before trouble occurs. The equipment consists of a small tank filled with anti-freeze solution into which is immersed a U-shaped highly polished gold plated tube. Through this tube passes a continuous flow of natural gas from a by-pass valve in the main pipe line. Both ends of the tube are provided with plate-glass windows. Above the windows, a General Electric light source and photoelectric relay are mounted so that the light shines through the tube and is reflected to the phototube. For controlling the temperature of the antifreeze solution, the tank is cooled with coils of a small domestic refrigerator and heated with suitable strip heaters. As the cooling system reduces the temperature of the solution, the gas is chilled until a film of condensate is deposited on the inside of the highly polished tube, thus reducing the reflection of light to the phototube. Being sensitive to a change of $\frac{1}{2}$ foot-candle, the phototube operates to disconnect the cooling coils and connect the heaters to warm the solution. When the solution is warm enough the condensate disappears and the process is repeated. A recording bulb type thermometer immersed in the solution records the temperature of the solution. On the same chart the natural gas pressure is also recorded. From these two records the dew-point is calculated. The apparatus was conceived and constructed by W. R. Beardsley and L. F. Carter of the Colorado Interstate Gas Company.

R. H. O.

SOME IMPROVEMENTS IN PRACTICAL FOURIER ANALYSIS AND THEIR APPLICATION TO X-RAY SCATTERING FROM LIQUIDS.

BY

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INTRODUCTION.

One of the more recent applications of Fourier analysis occurs in the quantitative investigation of liquids by x-rays following the theory of Zernike and Prins.[1] Although the following method was developed for this application, it is equally applicable to any problem requiring a Fourier analysis. If a modern mechanical analyzer (e.g. Henrici [2] or Michelson [3]) is available, the evaluation of a Fourier integral presents no difficulty. It is our purpose to show that, for occasional analyses at least, one need not depend upon such costly instruments, even when the required number of coefficients is very large. Like all other arithmetical methods we make use of the symmetry of the trigonometric functions in the four quadrants of a circle. The great reduction in the number of operations, which this allows, has been pointed out by Runge.[4] Since, however, the labor varies approximately as the square of the number of ordinates, the available standard forms become impractical for a large number of coefficients. We shall show that, by a certain transformation process, it is possible to double the number of ordinates with only slightly more than double the labor.

In the technique of numerical analysis the following improvements suggested by Lanczos were used: (1) a simple matrix scheme for any even number of ordinates can be used in place of available standard forms; (2) a transposition of odd ordinates into even ordinates reduces an analysis for $2n$ co-

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[1] Zernike and Prins, *Zeits. f. Phys.*, **41**, 184 (1927); *Zeits. f. Phys.*, **56**, 617 (1929).

[2] Henrici, *Phil. Mag.*, **38**, 110 (1894).

[3] Michelson and Stratton, *Phil. Mag.*, **45**, 85 (1898).

[4] Runge, *Zeit. für Math. und Physik*, **48**, 443 (1903); *Zeit. für Math. und Physik*, **53**, 117 (1905).

efficients to two analyses for n coefficients; (3) by using intermediate ordinates it is possible to estimate, before calculating any coefficients, the probable accuracy of the analysis; (4) any intermediate value of the Fourier integral can be determined from the calculated coefficients by interpolation. The first two improvements reduce the time spent in calculation and the probability of making errors, the third tests the accuracy of the analysis, and the fourth improvement allows the transform curve to be constructed with arbitrary exactness. Adopting these improvements the approximate times for Fourier analyses are: 10 minutes for 8 coefficients, 25 minutes for 16 coefficients, 60 minutes for 32 coefficients, and 140 minutes for 64 coefficients. Interpolation for an intermediate point requires 5 to 10 minutes.

IMPROVEMENTS IN PRACTICAL FOURIER ANALYSIS.

1. Introduction.

Let $y = f(x)$ be a finite, single-valued and continuous function in the range $-\pi$ to π. This function may be expanded into the infinite Fourier series

$$f(x) = \tfrac{1}{2}a_0 + a_1 \cos x + a_2 \cos 2x + \cdots + b_1 \sin x + b_2 \sin 2x + \cdots, \quad (1)$$

where

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos kx\, dx, \quad (2)$$

$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin kx\, dx. \quad (3)$$

The arithmetical method of evaluating the coefficients makes use of a finite number of ordinates y_k instead of the entire curve. The number of ordinates required depends solely on the convergence of the Fourier series and cannot be decided by merely looking at the graph of $f(x)$. Mathematically an infinite number of ordinates is required for the evaluation of the coefficients a_k and b_k. This, however, is only the case if $f(x)$ shall be reproduced with absolute accuracy. If $f(x)$ is to be reproduced only within the experimental errors, the situation is greatly simplified. The number of ordinates required for the evaluation of the coefficients need not exceed the number of terms which are practically present in the Fourier analysis of $f(x)$.[5] This means that within the limit of the expected accuracy $f(x)$ can be replaced by a finite sum of the form (1). If this sum contains, let us say, 21 cosine terms and 20 sine terms, then 41 readings will be sufficient for evaluation of the a_k and b_k. Since, however, we do not know in advance how many terms of the series have to be taken into consideration, we might better employ a larger number, say 50 readings. We could then calculate the coefficients beyond the twentieth overtone and find by calculation that the amplitudes of the higher overtones are negligible. An absolute guarantee can never be given, however, since it is possible that some overtones of the series drop out although some higher overtones are still present. Hence it is desirable to corroborate by some additional tests the evidence obtained by the vanishing of the last evaluated coefficients. In a later section two independent tests will be described which allow us to decide whether or not the number of readings was chosen sufficiently high. Even these tests, however, have only plausibility value and cannot be construed as an absolute guarantee for the lack of higher overtones. It is advisable to start the analysis with these two tests before beginning the numerical work which leads to the complete set of Fourier coefficients.

If after completing an analysis the number of coefficients is found to be too small, one can make use of the work already done by doubling the number of ordinates. The operations with the old ordinates remain unchanged and we have to add only the operations with the new ordinates. The additional labor is only slightly greater than that required for the original analysis and we now have twice the number of harmonic components as well as more accurate values for the previous components.

2. The Multiplication Matrix.

The principle of trigonometric interpolation is to find a finite series

$$\tfrac{1}{2}a_0 + a_1 \cos x + \cdots + a_{2n-1} \cos (2n - 1)x + \tfrac{1}{2}a_{2n} \cos 2nx \\ + b_1 \sin x + \cdots + b_{2n-1} \sin (2n - 1)x, \qquad (4)$$

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[5] Lanczos, *Journal of Math. and Phys.*, 17, 135 (1938).

which shall fit the given function $f(x)$ in the $4n$ equidistant points

$$x_k = k\frac{x}{2n} \quad (k = 0, \pm 1, \pm 2, \cdots \pm 2n). \tag{5}$$

Owing to the periodicity condition $y_{2n} = y_{-2n}$. The number of readings has been chosen as a multiple of 4 in order to get full benefit of the symmetry properties of the trigonometric functions in the 4 quadrants of the circle. The origin is put in the middle of the range. The details of the numerical work are discussed in Runge's book.[6]

The analysis is first divided into an even function (cosine analysis) and an odd function (sine analysis) by taking sums and differences of the original ordinates. Let the $4n$ equidistant ordinates be

$$y_{-(2n-1)} \quad y_{-(2n-2)} \quad \cdots \quad y_{-1} \quad y_0 \quad y_1 \quad \cdots \quad y_{2n-1} \quad y_{2n}. \tag{6}$$

They are arranged in reverse order in the following manner and the sums and differences obtained as shown

$$\begin{array}{lcccccc} & y_{2n} & y_{2n-1} & y_{2n-2} & \cdots & y_1 & y_0 \\ & & y_{-(2n-1)} & y_{-(2n-2)} & \cdots & y_{-1}\swarrow & \\ \hline \text{Sums} & u_{2n} & u_{2n-1} & u_{2n-2} & \cdots & u_1 & u_0 \\ \text{Differences} & & v_{2n-1} & v_{2n-2} & \cdots & v_1 & \end{array} \tag{7}$$

For example let $n = 4$ and the 16 equidistant ordinates be denoted as

$$y_{-7} \quad y_{-6} \quad \cdots \quad y_{-1} \quad y_0 \quad y_1 \quad \cdots \quad y_7 \quad y_8. \tag{8}$$

The sums and differences are obtained by writing the ordinates in reverse order and in a double row, the first going from left to right and the second from right to left

$$\begin{array}{lcccccc} & y_8 & y_7 & y_6 & \cdots & y_1 & y_0 \\ & & y_{-7} & y_{-6} & \cdots & y_{-1}\swarrow & \\ \hline \text{Sums} & u_8 & u_7 & u_6 & \cdots & u_1 & u_0 \\ \text{Differences} & & v_7 & v_6 & \cdots & v_1 & \end{array} \tag{9}$$

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[6] Runge and Koenig, "Vorlesungen über Numerisches Rechnen" (Springer, Berlin, 1924), page 211.

Then u_k enters only in the cosine and v_k only in the sine analysis. The coefficients of the complete harmonic analysis are given by the relations

$$a_k = \frac{2}{2n} A_k \tag{10}$$

and

$$b_k = \frac{2}{2n} B_k, \tag{11}$$

where

$$A_k = \sum_{\alpha=0}^{2n} u_\alpha \cos \frac{\pi}{2n} k\alpha \quad (k = 0, 1, 2, \cdots 2n) \tag{12}$$

and

$$B_k = \sum_{\alpha=1}^{2n-1} v_\alpha \sin \frac{\pi}{2n} k\alpha \quad (k = 1, 2, \cdots (2n - 1)). \tag{13}$$

The problem is to evaluate these sums A_k and B_k with a minimum amount of computation. To do this it is advantageous to pair the ordinates once more.

This is possible on the basis that:

$$\left.\begin{matrix}\cos\\ \sin\end{matrix}\right\} \frac{\pi}{2n} k(2n - \alpha) = \left.\begin{matrix}-\cos\\ \sin\end{matrix}\right\} \frac{\pi}{2n} k\alpha \text{ for } k \text{ odd,}$$

$$\left.\begin{matrix}\cos\\ \sin\end{matrix}\right\} \frac{\pi}{2n} k(2n - \alpha) = \left.\begin{matrix}\cos\\ -\sin\end{matrix}\right\} \frac{\pi}{2n} k\alpha \text{ for } k \text{ even.}$$

$$\begin{array}{lcccccc} & u_0 & u_1 & u_2 & \cdots & u_{n-1} & u_n \\ & u_{2n} & u_{2n-1} & u_{2n-2} & \cdots & u_{n+1} & \swarrow \\ \cline{2-7} \text{Sums} & u_0'' & u_1'' & u_2'' & \cdots & u_{n-1}'' & u_n'' \\ \text{Differences} & u_1' & u_1' & u_2' & \cdots & u_{n-1}' & \end{array} \tag{14}$$

$$\begin{array}{lccccc} & v_1 & v_2 & \cdots & v_{n-1} & v_n \\ & v_{2n-1} & v_{2n-2} & \cdots & v_{n+1} & \swarrow \\ \cline{2-6} \text{Sums} & v_1' & v_2' & \cdots & v_{n-1}' & v_n' \\ \text{Differences} & v_1'' & v_2'' & \cdots & v_{n-1}'' & \end{array} \tag{15}$$

Referring to our example in which $n = 4$ (16 ordinates) we have

$$\begin{array}{lccccc} & u_0 & u_1 & u_2 & u_3 & u_4 \\ & u_8 & u_7 & u_6 & u_5 \swarrow & \\ \cline{2-6} \text{Sums} & u_0'' & u_1'' & u_2'' & u_3'' & u_4'' \\ \text{Differences} & u_0' & u_1' & u_2' & u_3' & \end{array} \qquad (16)$$

$$\begin{array}{lcccc} & v_1 & v_2 & v_3 & v_4 \\ & v_7 & v_6 & v_5 \swarrow & \\ \cline{2-5} \text{Sums} & v_1' & v_2' & v_3' & v_4' \\ \text{Differences} & v_1'' & v_2'' & v_3'' & \end{array} \qquad (17)$$

The formulæ for the evaluation of the Fourier coefficients now become

$$A_k = \sum_{\alpha=1}^{n} \left. \begin{matrix} u_\alpha' \\ u_\alpha'' \end{matrix} \right\} \cos \frac{\pi}{2n} k\alpha \; \begin{matrix} (k = 1, 3, \cdots 2n - 1) \\ (k = 0, 2, \cdots 2n), \end{matrix} \qquad (18)$$

$$B_k = \sum_{\alpha=1}^{n} \left. \begin{matrix} v_\alpha' \\ v_\alpha'' \end{matrix} \right\} \sin \frac{\pi}{2n} k\alpha \; \begin{matrix} (k = 1, 3, \cdots 2n - 1) \\ (k = 2, 4, \cdots 2n - 2). \end{matrix} \qquad (19)$$

For the *odd* coefficients the *primed* and for the *even* coefficients the *double primed* quantities are employed. Hence by separating the even and odd coefficients we have reduced the range of the sum index α from $2n$ to n.

For our example the 8 Fourier coefficients for the cosine analysis are given by $a_k = \frac{2}{8} A_k$ where

$$A_k = \sum_{\alpha=0}^{4} \left. \begin{matrix} u_\alpha' \\ u_\alpha'' \end{matrix} \right\} \cos \frac{\pi}{8} k\alpha \; \begin{matrix} (k = 1, 3, 5, 7) \\ (k = 0, 2, 4, 6, 8), \end{matrix} \qquad (20)$$

$$B_k = \sum_{\alpha=1}^{4} \left. \begin{matrix} v_\alpha' \\ v_\alpha'' \end{matrix} \right\} \sin \frac{\pi}{8} k\alpha \; \begin{matrix} (k = 1, 3, 5, 7) \\ (k = 2, 4, 6). \end{matrix} \qquad (21)$$

However, the range of the fixed index k still extends until $2n$. If we separate the even and odd values of the sum index α, then a reduction to n is also possible here. Thus we define the following set of quantities

$$A_k' = \sum_\alpha \left.\begin{matrix} u_\alpha' \\ u_\alpha'' \end{matrix}\right\} \cos \frac{\pi}{2n} k\alpha \quad (\alpha = 1, 3, 5, \cdots n - 1), \tag{22}$$

$$A_k'' = \sum_\alpha \left.\begin{matrix} u_\alpha' \\ u_\alpha'' \end{matrix}\right\} \cos \frac{\pi}{2n} k\alpha \quad (\alpha = 0, 2, 4, \cdots n), \tag{23}$$

$$B_k' = \sum_\alpha \left.\begin{matrix} v_\alpha' \\ v_\alpha'' \end{matrix}\right\} \sin \frac{\pi}{2n} k\alpha \quad (\alpha = 1, 3, 5, \cdots n - 1), \tag{24}$$

$$B_k'' = \sum_\alpha \left.\begin{matrix} v_\alpha' \\ v_\alpha'' \end{matrix}\right\} \sin \frac{\pi}{2n} k\alpha \quad (\alpha = 2, 4, 6, \cdots n). \tag{25}$$

In these formulæ the index k runs only from 0 to n for the A_k and from 1 to n for the B_k quantities. The calculation of these sums gives all the coefficients directly by the formulæ

$$A_k = A_k'' + A_k', \qquad A_{2n-k} = A_k'' - A_k' \quad (k = 0, 1, 2, \cdots n), \tag{26}$$

$$B_k = B_k' + B_k'', \qquad B_{2n-k} = B_k' - B_k'' \quad (k = 1, 2, \cdots n). \tag{27}$$

In our example the 16 coefficients are determined by the following 16 simple sums

$$A_k' = u_1' \cos k\frac{\pi}{8} + u_3' \cos 3k\frac{\pi}{8} \quad (k = 1, 3)$$

$$A_k' = u_1'' \cos k\frac{\pi}{8} + u_3'' \cos 3k\frac{\pi}{8} \quad (k = 0, 2), \tag{28}$$

$$A_k'' = u_0' + u_2' \cos 2k\frac{\pi}{8} + u_4' \cos 4k\frac{\pi}{8} \quad (k = 1, 3)$$

$$A_k'' = u_0'' + u_2'' \cos 2k\frac{\pi}{8} + u_4'' \cos 4k\frac{\pi}{8} \quad (k = 0, 2, 4), \tag{29}$$

$$B_k' = v_1' \sin k\frac{\pi}{8} + v_3' \sin 3k\frac{\pi}{8} \quad (k = 1, 3)$$

$$B_k' = v_1'' \sin k\frac{\pi}{8} + v_3'' \sin 3k\frac{\pi}{8} \quad (k = 2, 4), \tag{30}$$

$$B_k'' = v_2' \sin 2k\frac{\pi}{8} + v_4' \sin 4k\frac{\pi}{8} \quad (k = 1, 3)$$

$$B_k'' = v_2'' \sin 2k\frac{\pi}{8} \qquad (k = 2). \tag{31}$$

The 9 coefficients of the cosine analysis are

$$A_k = A_k'' + A_k', \qquad A_{8-k} = A_k'' - A_k' \qquad (k = 0, 1, 2, 3, 4). \quad (32)$$

The 7 coefficients of the sine analysis are

$$B_k = B_k' + B_k'', \qquad B_{8-k} = B_k' - B_k'' \qquad (k = 1, 2, 3, 4). \quad (33)$$

From this scheme we see that the problem of calculation reduces to a small number of multiplications ($u_\alpha \cos \alpha k(\pi/8)$) and additions.

The practical evaluation of the Fourier coefficients is greatly facilitated by the application of a matrix scheme. All the cosine coefficients can be evaluated by one matrix, and all the sine coefficients by another matrix. Both matrices are constructed along similar lines. We describe first the matrix with the help of which the cosine coefficients can be evaluated.

Cosine Matrix.

We use the integers 0, 1, 2, 3, $\cdots$ n as a symbolic notation. An arbitrary integer k shall stand for $\cos k(\pi/2n)$. We first write down the following guiding line:

$$0, 1, 2, \cdots n-1, n, -(n-1), \cdots -1, -0, -1, \cdots -(n-1), n, n-1, \cdots 1. \quad (34)$$

This line represents a complete cycle of $\cos k(\pi/2n)$ values, k running from 0 to $4n - 1$. We now construct a matrix of $n + 1$ rows and $n + 1$ columns. We start with the row "zero," by putting down zero $n + 1$ times. Then the row "one" follows, by writing down the numbers of the guiding line: 0, 1, $\cdots$, until n. Then the row "two" follows by selecting every second number of the guiding line. Then the row "three" follows by selecting every third number of the guiding line, starting always with zero. Thus we continue, until the row "n" is reached. Whenever we reach the end of the line, we jump back to the beginning, and continue.

Example: $n = 4$.
Guiding line:

$$0,\ 1,\ 2,\ 3,\ 4,\ -3,\ -2,\ -1,\ -0,\ -1,\ -2,\ -3,\ 4,\ 3,\ 2,\ 1. \qquad (35)$$

Cosine Matrix ($n = 4$).

$$\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 2 & 4 & -2 & -0 \\ 0 & 3 & -2 & -1 & 4 \\ 0 & 4 & -0 & 4 & 0 \end{array} \qquad (36)$$

The next step is the separation of even and odd coefficients. We break the constructed matrix into 4 smaller matrices, which are characterized as follows:

(*a*) Even rows, Even columns
(*b*) Even rows, Odd columns
(*c*) Odd rows, Even columns
(*d*) Odd rows, Odd columns.

The "zero" is considered as one of the even numbers.

Example: $n = 4$.
For the example above we get the following 4 smaller matrices:

$$(a)\ \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 4 & -0 \\ 0 & -0 & 0 \end{array} \qquad (b)\ \begin{array}{cc} 0 & 0 \\ 2 & -2 \\ 4 & 4 \end{array}$$

$$(c)\ \begin{array}{ccc} 0 & 2 & 4 \\ 0 & -2 & 4 \end{array} \qquad (d)\ \begin{array}{cc} 1 & 3 \\ 3 & -1 \end{array} \qquad (37)$$

Since the actual value of the symbol "4" is zero, the third row of (*b*) and the third column of (*c*) can be omitted.

Each one of these matrices has to be used in connection with a proper set of ordinates:

(*a*) uses the even u'' (i.e. u_0'', u_2'', u_4''),
(*b*) uses the odd u'' (i.e. u_1', u_3'),
(*c*) uses the even u' (i.e. u_0', u_2'),
(*d*) uses the odd u' (i.e. u_1', u_3').

For the actual multiplications it is convenient to set the proper u-values at the top of the matrices, and to have at hand a table which gives the numerical values of the integer symbols. In the present example this table would be:

$$\begin{aligned} 0 &= \cos 0\frac{\pi}{8} = 1, & 1 &= \cos\frac{\pi}{8} = 0.92388, \\ 2 &= \cos 2\frac{\pi}{8} = 0.70711, & 3 &= \cos 3\frac{\pi}{8} = 0.38268, \\ 4 &= \cos 4\frac{\pi}{8} = 0. \end{aligned} \tag{38}$$

The horizontal sums resulting from the four multiplication schemes are denoted as follows:

from (a): A'' with even subscripts (i.e. A_0'', A_2'', A_4''),
from (b): A'' with odd subscripts (i.e. A_1'', A_3''),
from (c): A' with even subscripts (i.e. A_0', A_2'),
from (d): A' with odd subscripts (i.e. A_1', A_3').

The final Fourier coefficients are then

$$a_k = \tfrac{1}{4}A_k \qquad \text{and} \qquad a_{8-k} = \tfrac{1}{4}A_{8-k},$$

where

$$\begin{aligned} A_k &= A_k' + A_k'' \quad (k = 0, 1, 2, 3, 4), \\ A_{8n-k} &= A_k'' - A_k' \quad (k = 0, 1, 2, 3). \end{aligned} \tag{39}$$

The complete analysis requires only six multiplications.

Sine Matrix.

We again use the integers 1, 2, $\cdots n$ as a symbolic notation. An arbitrary integer k shall stand for $\sin k(\pi/2n)$. The guiding line is written as follows:

$$\begin{aligned} &1, 2, \cdots n-1, n, n-1, \cdots 2, 1, 0, -1, -2, \\ &\qquad \cdots -(n-1), -n, -(n-1), \cdots -1, 0. \end{aligned} \tag{40}$$

This line represents a complete cycle of $\sin k(\pi/2n)$ values, k running from 1 to $4n$.

We construct a matrix of n rows and n columns. The "zero" row and column is now missing. Otherwise the construction proceeds exactly as before in the cosine case.

Example: $n = 4$.

Guiding line:

$$1, 2, 3, 4, 3, 2, 1, 0, -1, -2, -3, -4, -3, -2, -1, 0. \qquad (41)$$

Sine Matrix ($n = 4$).

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 2 & 4 & 2 & 0 \\ 3 & 2 & -1 & -4 \\ 4 & 0 & -4 & 0 \end{array} \qquad (42)$$

We separate the even and odd rows and columns as before:

(a) Even rows, Even columns
(b) Even rows, Odd columns
(c) Odd rows, Even columns
(d) Odd rows, Odd columns.

Example: $n = 4$.

$$(a)\ \begin{array}{cc} 4 & 0 \\ 0 & 0 \end{array} \qquad (b)\ \begin{array}{cc} 2 & 2 \\ 4 & -4 \end{array}$$

$$(c)\ \begin{array}{cc} 2 & 4 \\ 2 & -4 \end{array} \qquad (d)\ \begin{array}{cc} 1 & 3 \\ 3 & -1 \end{array} \qquad (43)$$

(a) uses the even v'' (i.e. v_2'', v_4''),
(b) uses the odd v'' (i.e. v_1'', v_3''),
(c) uses the even v' (i.e. v_2', v_4'),
(d) uses the odd v' (i.e. v_1', v_3').

We put the proper v-values at the top of the matrices. Moreover, we have to have a table at hand which gives the values of the integer symbols. In the present example this table would look as follows:

$$0 = \sin 0\frac{\pi}{8} = 0, \qquad 1 = \sin\frac{\pi}{8} = 0.38268,$$

$$2 = \sin 2\frac{\pi}{8} = 0.70711, \qquad 3 = \sin 3\frac{\pi}{8} = 0.92388,$$

$$4 = \sin 4\frac{\pi}{8} = 1. \tag{44}$$

The results of the 4 multiplication schemes are denoted as follows:

from (*a*): B'' with even subscripts (i.e. B_2'', B_4''),
from (*b*): B'' with odd subscripts (i.e. B_1'', B_3''),
from (*c*): B' with even subscripts (i.e. B_2', B_4'),
from (*d*): B' with odd subscripts (i.e. B_1', B_3').

The final Fourier coefficients are then

$$b_k = \tfrac{1}{4}B_k \qquad \text{and} \qquad b_{8-k} = \tfrac{1}{4}B_{8-k},$$

where

$$B_k = B_k' + B_k'' \quad (k = 1, 2, 3, 4)$$

and

$$B_{8n-k} = -B_k'' + B_k' \quad (k = 1, 2, 3). \tag{45}$$

Again the complete analysis requires only six multiplications.

3. The Transformation of Ordinates.

Any arithmetical scheme for harmonic analysis, including the matrix scheme, becomes increasingly complicated as the number of required coefficients increases. The number of operations involved in an analysis for $2n$ coefficients is approximately four times the number of operations for n coefficients. We shall now describe a method which eliminates the necessity of complicated schemes by reducing either a cosine or a sine analysis for $4n$ coefficients to two analyses for $2n$ coefficients. If desired, this reduction process can be applied twice or three times.

The method is based upon the following observation. Let us select every second column of the multiplication matrix;

that is, let us assign only even values to the sum index α.

$$\alpha = 0, 2, 4, 6, \cdots 4n. \tag{46}$$

If we now put

$$\alpha = 2\alpha' \quad (\alpha' = 0, 1, 2, \cdots 2n), \tag{47}$$

we obtain

$$\frac{\pi}{4n}\alpha k = \frac{\pi}{2n}\alpha' k, \tag{48}$$

which shows that the contribution of the even ordinates to a Fourier analysis of $4n$ coefficients is exactly the same as the contribution of all the ordinates to a Fourier analysis of $2n$ coefficients. In regard to the contribution of even ordinates the matrix scheme for one-half the number of ordinates can replace the matrix scheme for the total number of ordinates.

We now wish to show that the contribution of the odd ordinates is also reducible to the same matrix scheme for one-half the number of ordinates. We will then be able to make an analysis for $4n$ coefficients by using twice the matrix scheme for $2n$ coefficients; once to get the contribution of the even ordinates, and once to get the contribution of the odd ordinates. This is possible by means of a transformation process introducing a phase difference, which transposes the odd ordinates, as far as their contribution to the Fourier coefficients is concerned, to the place of the even ordinates.

The contribution of the odd ordinates is, from equations (12) and (13)

$$A_k' = \sum_{\beta=1}^{4n-1} u_\beta \cos\frac{\pi}{4n}k\beta \quad (\beta = 1, 3, 5, \cdots 4n - 1), \tag{49}$$

$$B_k' = \sum_{\beta=1}^{4n-1} v_\beta \sin\frac{\pi}{4n}k\beta \quad (\beta = 1, 3, 5, \cdots 4n - 1). \tag{50}$$

Let $\beta = 2\alpha + 1$ where

$$\alpha = 0, 1, 2, \cdots 2n - 1. \tag{51}$$

Then

$$A_k' = \sum_{\alpha=0}^{2n-1} u_{2\alpha+1} \cos\frac{\pi}{2n}k(\alpha + \tfrac{1}{2}), \tag{52}$$

$$B_k' = \sum_{\alpha=1}^{2n-1} v_{2\alpha+1} \sin\frac{\pi}{2n}k(\alpha + \tfrac{1}{2}). \tag{53}$$

We now define a new set of u_k and v_k ordinates called the transformed ordinates and denote these by $\bar{u}_k$ and $\bar{v}_k$. They are defined by the equations

$$\begin{array}{llllllll}
u_1 & = 2\bar{u}_0 & + \bar{u}_1 & & v_1 & = 2\bar{v}_0 & - \bar{v}_1 & \\
u_3 & = \bar{u}_1 & + \bar{u}_2 & & v_3 & = \bar{v}_1 & - \bar{v}_2 & \\
u_5 & = \bar{u}_2 & + \bar{u}_3 & & v_5 & = \bar{v}_2 & - \bar{v}_3 & \\
\cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & \\
\cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & \\
\cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & \\
u_{2k+1} & = \bar{u}_k & + \bar{u}_{k+1} & & v_{2k+1} & = \bar{v}_k & - \bar{v}_{k+1} & \\
\cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & \\
\cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & \\
\cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & \\
u_{4n-3} & = \bar{u}_{2n-2} & + \bar{u}_{2n-1} & & v_{4n-3} & = \bar{v}_{2n-2} & - \bar{v}_{2n-1} & \\
u_{4n-1} & = \bar{u}_{2n-1} & + \bar{u}_{2n} & & v_{4n-1} & = \bar{v}_{2n-1} & - \bar{v}_{2n} & \\
\bar{u}_{2n} & = 0 & & & \bar{v}_{2n} & = 0 & &
\end{array} \tag{54}$$

The evaluation of the transformed ordinates is a quick and simple process. As seen from (54) $u_{4n-1} = \bar{u}_{2n-1}$. On starting with the last ordinate $u_{2n} = 0$ we proceed successively from right to left.

Consider the case $2n = 8$ (i.e. 16 ordinates). The transforming process for the odd ordinates can be carried out in a scheme like the following:

$$\begin{array}{llllll}
 & u_1 & u_3 & u_5 & u_7 & \\
 & \bar{u}_1 \nwarrow & \bar{u}_2 \nwarrow & \bar{u}_3 \nwarrow & 0 \nwarrow & \\
\hline
\text{Differences} & 2\bar{u}_0 & \searrow \bar{u}_1 & \searrow \bar{u}_2 & \searrow \bar{u}_3 & \searrow \bar{u}_4 = 0 \\
 & & & & & \\
 & v_1 & v_3 & v_5 & v_7 & \\
 & \bar{v}_1 \nwarrow & \bar{v}_2 \nwarrow & \bar{v}_3 \nwarrow & 0 \nwarrow & \\
\hline
\text{Sums} & 2\bar{v}_0 & \searrow \bar{v}_1 & \searrow \bar{v}_2 & \searrow \bar{v}_3 & \searrow \bar{v}_4 = 0
\end{array} \tag{55}$$

To check the calculation we have

$$\begin{aligned}
2\bar{u}_0 &= u_1 - u_3 + u_5 - u_7 \\
2\bar{v}_0 &= v_1 + v_3 + v_5 + v_7.
\end{aligned} \tag{56}$$

Owing to the definition of the transformed ordinates $\bar{u}_k$ and $\bar{v}_k$ in equations (54) the contribution of the odd ordinates becomes, by substitution in (52) and (53)

$$A_k' = 2\bar{u}_0 \cos\frac{\pi}{4n}k + \bar{u}_1\left(\cos\frac{\pi}{4n}k + \cos\frac{\pi}{4n}3k\right) + \bar{u}_2\left(\cos\frac{\pi}{4n}3k + \cos\frac{\pi}{4n}5k\right) + \cdots \tag{57}$$

$$= 2\cos\frac{\pi}{4n}k \sum_{\alpha=0}^{2n} \bar{u}_\alpha \cos\frac{\pi}{2n}k\alpha,$$

$$B_k' = 2\bar{v}_0 \sin\frac{\pi}{4n}k + \bar{v}_1\left(\sin\frac{\pi}{4n}3k - \sin\frac{\pi}{4n}k\right) + \bar{v}_2\left(\sin\frac{\pi}{4n}5k - \sin\frac{\pi}{4n}3k\right) + \cdots \tag{58}$$

$$= 2\sin\frac{\pi}{4n}k \sum_{\alpha=0}^{2n} \bar{v}_\alpha \cos\frac{\pi}{2n}k\alpha.$$

Thus we see that, apart from the weight factors $2\cos(\pi/4n)k$ and $2\sin(\pi/4n)k$, the calculation with the transformed ordinates is identical to the cosine analysis of half the number of ordinates. The final coefficients of the analysis are then given by equations (26) and (27).

For example, let us consider again the case of 32 ordinates. These y_k are paired and reduced, as outlined previously, to u_k requiring a cosine analysis for 16 coefficients and to v_k requiring a sine analysis for 16 coefficients.

The even u_k are analyzed directly using the cosine scheme for 8 coefficients. The odd u_k are transformed, analyzed by the cosine scheme for 8 coefficients, and multiplied by the weight factor $2\cos(\pi/16)k$. If we call the result of the analysis of the even u_k, A_k'', and of the odd u_k, A_k'; then the final cosine coefficients are

$$A_k = A_k'' + A_k' \quad \text{and} \quad A_{16-k} = A_k'' - A_k' \quad (k = 0, 1, 2, \cdots 8). \tag{59}$$

Similarly the even v_k are analyzed directly using the sine scheme for 8 coefficients. The odd v_k are transformed, analyzed by the cosine scheme for 8 coefficients, and multiplied by the weight factor $2\sin(\pi/16)k$. If we call the result of the analysis of the even v_k, B_k'', and of the odd v_k, B_k'; then the final sine coefficients are

$$B_k = B_k' + B_k'', \qquad B_{16-k} = B_k' - B_k'' \qquad (k = 0, 1, 2, \cdots 8). \qquad (60)$$

For 64 ordinates we might apply the reduction process twice. The cosine analysis for 32 coefficients is then accomplished by making 4 cosine analyses for 8 coefficients, and the sine analysis for 32 coefficients by making 1 sine and 3 cosine analyses for 8 coefficients.

The great advantage of the transformation process is its ability to reduce large matrices (or complicated standard forms) to small matrices (or simple standard forms). For analyses requiring a large number of coefficients this reduction more than compensates for the small amount of labor involved in the transformation process and in the multiplication by weight factors. In addition, the transformation process makes it much easier to check the analyses and to locate errors in computation. Finally, if we have made an analysis for $2n$ coefficients and find the convergence unsatisfactory we can double the number of coefficients by making one additional analysis for $2n$ coefficients. In this way we obtain the full benefit of our original calculations.

(*To be continued in May issue.*)

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

STRUCTURAL PROTECTION AGAINST INCENDIARY BOMBS.

The hazard from all types of incendiary bombs can be decreased by applying an incombustible covering over the attic or upper floor, assuming that this will act as a stopping layer for at least some of the bombs that may be used. While, without defining type and weight, nothing definite can be said relative to the penetration of these bombs, even the lighter ones would be expected to go through an ordinary wood frame roof with the usual roof coverings and possibly also an attic or upper floor.

A covering applied to such floors would increase the resistance to penetration, besides affording protection against ignition by the bomb. The thickness to be applied need be limited only by practical considerations of cost and the weight that the construction can support. There will be progressive gain in safety with increase in thickness. A 1 to 1½ inch thickness of portland cement mortar or concrete, cast in place or as precast slabs, will prevent ignition of a wood floor from a bomb such as one of the kilogram thermite-magnesium type. Lesser thicknesses would give temporary protection and the incombustible finish would assist materially in extinguishing or removing the bomb without ignition of the floor beneath.

Slabs made with cinders, slag, and other light-weight aggregates are somewhat more resistive than those made with sand and gravel, although the latter do not disintegrate appreciably under the heat of the bomb. Precast slabs may require a finish consisting of portland cement mortar, or magnesite to give an even wearing surface. The wearing properties of portland cement mortar can be greatly increased by the addition of emulsified asphalt in the mix. The precast slabs should preferably have beveled edges to enable the mortar to fill the joints fully, and can be nailed in place through such joints.

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* Communicated by the Director.

An alternative to nailing might be to bed them in mortar. Where the traffic is light and surfaces are fairly even, it might serve the purpose to fill the joints with mortar and apply a rubber-base concrete paint in lieu of other top finish.

Slabs of burnt clay are another possibility—a development that deserves the attention of the clay products industries. Present stock sizes in solid or near solid units of thickness from 2 inches up might be used if other considerations permit.

Where the ceiling construction has no floor above, the most practical method would probably be to floor it over with boards (not necessarily matched), on which the covering is applied.

However, the bomb is likely to ignite any combustible contents near it. As protection, the side walls and roof rafters can be covered with boards made largely or wholly of incombustible materials, such as cement-asbestos boards or gypsum wallboards, or with plaster on metal or gypsum lath. Some degree of protection can also be obtained with heavy coats of sodium silicate, whitewash, borax paints, or other fire retardant coatings applied with brush or spray. Spaces along the walls into the construction beneath should be blocked with the floor covering material or other incombustible fill.

Protections can also be applied on wood-frame flat roofs, but preferably should be not less than 3 or 4 inches thick to give the needed additional resistance to penetration required for a stopping layer in this location. The advantage within limits of using light-weight materials is apparent in decreasing the load on the structure and increasing heat insulation, particularly for ordinary temperature conditions. The protecting layer should be anchored into masonry or other exterior wall construction, and be waterproofed or otherwise covered to form a weather-tight covering. Sloping roofs require less thickness but introduce greater difficulties in application.

Between the stopping layer and any refuge room a floor should intervene, equivalent in impact resistance to a single-board wood floor. This gives additional resistance to penetration and to impacts from falling materials in case of disruption of the construction under the stopping layer.

EVALUATION OF PROCESSES FOR THE PRODUCTION OF RUBBER.

The Bureau has outlined a procedure for evaluating the numerous plans which are being offered for the production of rubber in the present emergency. This procedure is essentially the same as that used by industry for determining the merit of any new process, and is equally applicable to proposals for growing, synthesizing, or reclaiming rubber.

According to this procedure, a potential investor, sponsor, or other party interested in finding out the commercial possibilities of a new process for obtaining rubber should take the following five steps:

1. An explicit, written description of the process should be obtained, together with a statement of the yield of rubber to be expected.
2. This description should be reviewed by a disinterested authority to determine whether or not it is consistent with known facts and accepted scientific principles.
3. If the process appears practical, it should be tested by having a competent laboratory repeat it independently of the inventor.
4. If the process yields rubber as claimed, samples should be tested to determine the quality and probable commercial value of the rubber.
5. If the foregoing laboratory tests are satisfactory the process should be operated on several successively larger scales until a scale is reached in which the product is made in commercial quantities.

Decision as to whether or not a process is to be used in large scale production should be reserved until all five of these steps have been passed successfully. This procedure will necessarily cost time and money and is quite at variance with the common notion that it is only necessary to test a small sample and if satisfactory results are obtained to proceed at once to start a plantation if the rubber is to be grown or to build a factory if it is to be made by synthesis or reclaiming. Actually such a hasty procedure would be likely to prove disastrous to even the best of processes. Some of the reasons back of the five steps which have been laid down are discussed in the following paragraphs.

Description of the Process.—An adequate, written description is a prime requisite for any intelligent consideration of a process. If a process is secret, or if it cannot be described so that it can be repeated without assistance from the inventor, it is not ready for use in production.

Many inventors have sought backing for secret processes by exhibiting samples, and often prospective investors have become so engrossed in testing samples as to lose sight of the fact that they have no evidence that the samples were produced as claimed. In order to meet objections on this score the promoters of secret processes sometimes arrange elaborate demonstrations in which a process can be witnessed without a disclosure being made. Numerous hoaxes have been perpetrated in connection with such demonstrations and hundreds of pounds of rubber have been obtained from carefully sealed, locked, and watched containers with seemingly as much ease as a magician brings a rabbit from a hat. It cannot be too strongly emphasized that there is no way in which a secret process can be adequately evaluated.

A statement of the yield is an essential part of the disclosure of a process, inasmuch as many of the ways in which rubber can be obtained are not practical because the yield is too small. Thousands of plants, for example, contain small amounts of rubber, and hundreds have been tried at one time or another as sources of rubber, but only a very few yield enough to warrant their being considered for use in production.

Review of the Process.—A critical review of proposed processes for obtaining rubber usually eliminates most of them from further consideration either because they are not new or because they are not based on sound scientific principles. The production of a rubber-like material from a vegetable oil and sulphur, for example, is often heralded as a new discovery in spite of the fact that the product, factice, has been known for a hundred years and is a common article of commerce.

Sometimes the description of a process involves reactions or procedures which seem fantastic and highly improbable to the chemist, but appear plausible to the layman. In such cases it may be advisable to proceed with a test of the process even though the chances that it will work are vanishingly small.

The claim is sometimes made that because a process has been patented it has been tried out and approved by the Government. Such, of course, is not the case, since a patent is designed simply to protect the inventor in his use of a process, but does not testify to its practicability.

Independent Repetition of the Process.—Independent repetition of the process under consideration is a sure safeguard against fraudulent and unworkable claims, and also serves to determine whether or not the process has been adequately and completely described. This work should be undertaken by competent chemists, engineers, plant physiologists, or other technically trained investigators as required by the nature of the process. They should follow written instructions furnished by the inventor, and should independently obtain and identify all materials used. Their work should give a definite answer to the question, "Does the process yield rubber, and if so, in what amount?"

Testing of the Rubber.—If the independent trial of the process yields rubber in practical amounts then, and only then, should serious consideration be given to testing the rubber to determine its quality and probable commercial value. Even at this stage the tests need not be highly critical since the requirements of the industry are so wide and varied that many different rubberlike products can find profitable use.

Development of Process to Commercial Stage.—If the rubber obtained on a laboratory scale is satisfactory both in quality and quantity, the process is still not ready for immediate large scale use but must go through a development stage in which the production is expanded step by step. Many difficulties are likely to be encountered in production which were not even anticipated from laboratory work. Expansion by stages enables these difficulties to be found and corrected and also aids in determining costs and estimating the size of units for the most efficient production.

This development procedure is equally necessary whether the rubber is to be collected wild from a hitherto unused plant, or grown on a plantation, or produced in a factory by synthesis or reclaiming. The processes which are now being pushed into large scale production to meet the present emergency are not new and untried but have all been in the development stage for a number of years and have a background of successful commercial use.

CALCIUM CARBONATE IN RUBBER.

A study of the effect of fillers on the electrical properties of rubber is being made at the Bureau in order to arrive at general relations between the properties and proportion of filler and the dielectric constant, power factor, and conductivity of the compounded rubber. The results obtained with calcium carbonate are reported by Arnold H. Scott and Archibald T. McPherson in the *Journal of Research* for March (RP1457). The system: rubber-calcium carbonate presents a relatively simple case for study because the electrical properties of calcium carbonate do not differ as greatly from those of rubber as do the properties of many other fillers.

The experimental compounds used in this investigation were prepared by incorporating reagent-quality calcium carbonate into two simple base compounds; one consisted only of vistanex, a synthetic rubber-like hydrocarbon, which was employed without vulcanization; the other was made from natural rubber and was vulcanized by means of tetramethylthiuram disulphide. The calcium carbonate was used in proportions up to about 50 per cent. by volume.

It was found possible to express the relation between the dielectric constant and the volume fraction of calcium carbonate by the general relation,

$$\frac{K - K_1}{K + FK_1} = V_2 \frac{K_2 - K_1}{K_2 + FK_1},$$

where F is a constant having the value 2.7; K, K_1 and K_2 are the dielectric constants of the mixture, the base compound, and the calcium carbonate, respectively; and V_2 is the volume fraction of the calcium carbonate. At a frequency of 1,000 cycles per second and a temperature of about 25° C. the natural rubber base compound has a dielectric constant 2.52, the vistanex, 2.29, and the calcium carbonate, 8.75. The power factor and the conductivity increase with increasing percentages of calcium carbonate, but do not seem to bear any simple relation to the composition.

TENSILE ELASTIC PROPERTIES OF METALS.

Sensitive tension tests made by Dunlap J. McAdam, Jr. and Russel W. Mebs of the Bureau's Metallurgy Division, upon specimens of nickel, copper, open-hearth iron and typical steels, have led to the identification of several basic factors which influence the variation of the elastic properties of metals during plastic extension. These factors are: (*a*) the "macroscopic" and "microstructural" internal stresses; (*b*) the work-hardening rate, which probably is directly associated with the permanent crystal lattice expansion of the metal occurring during deformation; and (*c*) the orientation of the metal crystal lattice.

As explained in the March *Journal of Research* (RP1459), the elastic properties measured were the "proof stresses," the modulus of elasticity, and the variation of the modulus with stress. The "proof stresses," i.e., stresses producing specified amounts of permanent deformation, were derived from curves of the variation of stress with permanent extension or set. The modulus of elasticity and its variation with stress were derived from curves of variation of stress with elastic deformation or strain. Diagrams are given showing the variation of the proof stresses and elastic indices with prior plastic extension.

A study of these diagrams and a survey of the literature permitted the identification of these basic factors. A fairly complete survey and discussion is given of the literature dealing with (*a*) crystal reorientation produced by plastic deformation and annealing, and (*b*) the various types of internal stress. Diagrams showing the directional variation of the tension and shear moduli and the "elastic limit" for single crystals of various metals are reproduced.

TEMPERATURE ESTIMATES OF THE PLANET MARS.

In earlier publications (Scientific Papers S512 and S553) data were given on the planetary radiation emanating from different parts of the surface of the planet Mars as measured by W. W. Coblentz, chief of the Bureau's Radiometry Section, with a thermocouple and filters mounted on the reflecting telescope of the Lowell Observatory at Flagstaff, Arizona. From these radiometric measurements, planetary temperatures were calculated by five methods, using the transmission data of the earth's atmosphere at 7 to 15 μ published by Fowle in the Smithsonian Physical Tables. In the meantime, based upon observations at Flagstaff (elevation 7,250 ft.), Adel and Lampland have published atmospheric transmissions that appear to be somewhat different from the Smithsonian measurements in the spectral band extending from 7 to 15 μ.

Since the planetary radiation measurements were made at Flagstaff, Doctor Coblentz has recalculated the data obtained during the oppositions of Mars in 1924 and in 1926, using the atmospheric transmission observed at that station. In the *Journal of Research* for March (RP1458), he reviews the five methods and procedures previously used by three calculators in deriving planetary temperatures, and shows that in four of these methods of calculation, which employ the law of total radiation of a black body, there is but little difference in the planetary temperatures deduced by using the transmission coefficients of the earth's atmosphere published by these two observatories. This is owing to the fortuitous circumstance that, while they differ in spectral outline, the total areas encompassed by these two transmission curves, and hence, the calculated total amounts of planetary radiation transmitted by the earth's atmosphere are nearly equal. Therefore no revision is made of the planetary temperatures previously deduced by four methods, involving the law of total radiation.

On the other hand, the fifth method, using the law of spectral radiation of a black body and the Flagstaff transmission coefficients of the atmosphere, gives planetary temperatures that are entirely different from the values previously deduced by this and by the other four methods using the Smithsonian data. This inconsistency can be eliminated, at least in part, and the data harmonized by taking into consideration selective emission of the planetary surface.

On the assumption that the surface of Mars, like the Earth, is composed of silicates (feldspar, mica, silica, etc.) which have a high selective emission at 8 to 10 μ, the temperatures derived from the spectral components of planetary radiation of Mars, measured in 1926, are in good agreement with the values calculated by other methods which are less affected by selective spectral emission. From this it would appear that, assuming the temperatures deduced by the other methods of calculation are indicative of meteorological conditions at the time of making the observations, the fluorite filter method of analysis may be a useful means of securing information on the selective spectral emission of the planetary surface.

In a supplementary note, a new thermocouple of bismuth and chromel-P wires, suitable for planetary radiometry, is described.

Process to Expand Magnesium Supply.—LESTER VELIE. (*The Journal of Commerce*, Vol. 191, No. 14763.) Production of magnesium, a metal that is one-third lighter and costs one-third less than aluminum, will be materially expanded as the result of a new process developed by the Mathieson Alkali Works. The process, developed by the Mathieson organization's engineers results not only in the production of magnesium but yields as well, as a by-product, the extremely valuable war essential, chlorine. Some 2.3 pounds of chlorine are produced with every pound of magnesium. The raw material used is Dolomite, a rock resembling limestone and consisting of limestone and magnesium carbonate. The material is found in generous quantities in Texas. The process involves the use of calcium chloride which is produced as a by-product in the manufacture of soda ash and ammonia. Dolomite is calcined, using natural gas for the purpose. The resulting oxides of calcium and magnesium are treated with the calcium chloride. The mass is then treated with carbon dioxide, obtained from the calcination of the dolomite. This converts the calcium into the insoluble carbonate, leaving magnesium chloride. The magnesium chloride, after being concentrated, is electrolyzed in a new type of cell developed by Mathieson in coöperation with the Consolidated Mining and Smelting Co. of Canada Ltd. This cell, known as the McMullin cell, is Mathieson's major contribution to the science of magnesium production. Experiments with this cell were conducted at Trail, British Columbia, the site of the Consolidated Mining and Smelting Co. plant. A pilot plant was built there and magnesium produced. Partnership of Mathieson and Consolidated Mining in the search for a new magnesium product grew naturally from Mathieson's experience in electro-chemical and chlorine manufacture and Consolidated's technical background in metals. The new process was achieved following a year of intensive research.

R. H. O.

THE FRANKLIN INSTITUTE.

STATED MEETING OF THE INSTITUTE WEDNESDAY, MARCH 18, 1942.

The stated monthly meeting of The Franklin Institute was opened at 8:15 o'clock on Wednesday, March eighteenth, by the playing of the national anthem, while the audience stood at attention.

The meeting was then called to order by the presiding officer, Mr. Charles S. Redding, President, who read the instructions regarding air raids and the application of rules of conduct during such raids.

The minutes of the previous meeting were approved as printed in the Journal of the Institute, March issue.

The President pointed out that one of the greatest values accompanying membership in The Franklin Institute is the use of the Library and reported that, for the use of members, a collection of material on civil defense, air-raid precautions, and present day warfare had been set aside for reference and use. He stated also, that books on camouflage, to be consulted by persons visiting the camouflage exhibit in the museum, had been gathered together in one place for ready reference. The Secretary reported the following additions to membership during February:

Active	48
Associate	50
Student	37
Total	135

He then extended a cordial invitation to all present to attend Medal Day Exercises on the afternoon of Wednesday, April 15, 1942, and expressed the hope that they would also attend the subscription dinner in the evening, given in honor of the medalists of the afternoon.

Upon recommendation of the Board of Managers, he presented for Honorary Membership in The Franklin Institute, custodian of the National Franklin Memorial, the Honorable George Wharton Pepper, "in consideration of his inspiring leadership in the movement to reincarnate in our times the homely virtues of our patron, Benjamin Franklin, thereby increasing human understanding and kindling anew courage, loyalty and patriotism."

This motion, duly seconded and carried, was unanimously passed.

Mr. Redding announced that the Certificate of Honorary Membership would be presented to Mr. Pepper on the evening of Medal Day, April fifteenth.

The President then introduced the speaker of the evening, Dr. W. F. G. Swann, Director, Bartol Research Foundation of The Franklin Institute, and pointed out that Dr. Swann was well known to the audience because of their familiarity with his work at the Bartol and through papers which had appeared in the JOURNAL.

Dr. Swann, taking as his subject "Report on the Work of the Bartol Research Foundation of The Franklin Institute," told of the activities carried on in the laboratories, stressing coöperation with the Government, of which little could be related. He then gave in detail the important work being done in the Geiger-Müller counter field and in cosmic rays. The lecture was illustrated by lantern slides.

The meeting adjourned at 9:45 with a rising vote of thanks to the lecturer.

March 19, 1942.

COMMITTEE ON SCIENCE AND THE ARTS.

(*Abstract of Proceedings of Stated Meeting held Wednesday, March 11, 1942.*)

HALL OF THE COMMITTEE,
PHILADELPHIA, MARCH 11, 1942.

MR. JOHN B. KLUMPP *in the Chair.*

The following report was presented for final action:

No. 3075: Cenco Hypervac 100 Vacuum Pump.

This report recommended the award of a Certificate of Merit to Carl S. Hornberger, of Chicago, Illinois, "In consideration of the excellence of design and construction of the Cenco Hypervac 100 Vacuum Pump, which has resulted in its wide recognition as a most useful piece of laboratory apparatus."

JOHN FRAZER,
Secretary to Committee.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from nine o'clock A.M. until five o'clock P.M., Wednesdays and Thursdays from two until ten o'clock P.M.

RECENT ADDITIONS.

AERONAUTICS.

COLVIN, FRED H. Aircraft Handbook. Fifth Edition. 1942.
SHIELDS, BERT A. Air Pilot Training. 1942.
SVENSEN, CARL LARS. A Manual of Aircraft Drafting. 1941.

ARCHITECTURE AND BUILDING.

American Concrete Institute. Proceedings. Volume 37. 1941.

American Society of Heating and Ventilating Engineers. Heating, Ventilating and Air Conditioning Guide. Volume 20. 1942.

BIOCHEMISTRY.

Annual Review of Biochemistry. Cumulative Index. Volumes 1–10. 1942.

CHEMISTRY AND CHEMICAL TECHNOLOGY.

Brown, Francis Wm. Chemical Problems and their Solutions. 1941.

Delmonte, J. Plastics in Engineering. Second Edition. 1942.

Institution of Chemical Engineers. Transactions. Volume 18. 1940.

Nelson, W. L. Petroleum Refinery Engineering. Second Edition. 1941.

Pierce, David E. Chemical Engineering for Production Supervision. First Edition. 1942.

Seidell, Atherton. Solubilities of Organic Compounds. Third Edition. Volume 2. 1941.

Thomas, Charles Allen. Anhydrous Aluminum Chloride in Organic Chemistry. 1941.

Vilbrandt, Frank C. Chemical Engineering Plant Design. Second Edition. 1942.

Welcher, Frank. Chemical Solutions. 1942.

233
12

DICTIONARIES, ALMANACS, ETC.

Fuller, Edmund, Editor. Thesaurus of Quotations. 1941.

World Almanac and Book of Facts for 1942.

EDUCATION.

Gray, George W. Education on an International Scale. 1941.

ELECTRICITY AND ELECTRICAL ENGINEERING.

Cobine, James Dillon. Gaseous Conductors. First Edition. 1941.

ENGINEERING.

American Society for Testing Materials. Proceedings of the Forty-Fourth Annual Meeting 1941. Volume 41. 1942.

Frocht, Max Mark. Photoelasticity. Volume 1. 1941.

Hoover, Theodore Jesse, and John Charles Lounsbury Fish. The Engineering Profession. 1941.

Marin, Joseph. Mechanical Properties of Materials and Design. First Edition. 1942.

Southwell, R. V. Relaxation Methods in Engineering Science. 1940.

Steinman, David B., and Sara Ruth Watson. Bridges and their Builders. 1941.

INDUSTRIAL MANAGEMENT.

Bethel, Lawrence L., and others. Production Control. First Edition. 1942.

National Research Council. Committee on Work in Industry. Fatigue of Workers. 1941.

RAUTENSTRAUCH, WALTER. The Design of Manufacturing Enterprises. 1941.

MANUFACTURES.

CLAPPERTON, ROBERT HENDERSON, AND WILLIAM HENDERSON. Modern Paper-Making. 1941.

DAVIES, A. C. Science and Practice of Welding. 1941.

MARSH, J. T. Mercerising. 1941.

MILHAM, WILLIS I. Time and Timekeepers. 1941.

MOBERG, IVAR. Cotton Loomfixers' Manual. First Edition. 1942.

PERRY, THOMAS D. Modern Plywood. 1942.

VAN DOREN, HAROLD. Industrial Design. 1940.

MECHANICAL ENGINEERING.

BUCKINGHAM, EARLE. Principles of Interchangeable Manufacturing. Second Edition. 1941.

COLVIN, FRED H. Gages. First Edition. 1942.

COUSINS, FRED M. Theory and Design of Springs. 1940.

KURREIN, MAX, AND F. C. LEA. Cutting Tools for Metal Machining. No Date.

METALLURGY.

SWEETSER, RALPH H. Blast Furnace Practice. First Edition. 1938.

METEOROLOGY.

WENSTROM, WILLIAM HOLMES. Weather and the Ocean of Air. 1942.

MILITARY ENGINEERING.

HICKS, JAMES E. What the Citizen Should Know About our Arms and Weapons. 1941.

JOHNSON, MELVIN M., JR., AND CHARLES T. HAVEN. Automatic Arms. 1941.

Museum of Modern Art, New York. Britain at War. 1941.

STROM, HANS. Wörterbuch für die Waffen-Munitions- und Sprengstoffindustrie. 2 Auflage. 1939.

THOMAS, JOSEPH MILLER. Elementary Mathematics in Artillery Fire. First Edition. 1942.

WESSMAN, HAROLD E., AND WILLIAM A. ROSE. Aerial Bombardment Protection. 1942.

MINERALOGY.

ROGERS, AUSTIN F., AND PAUL F. KERR. Optical Mineralogy. Second Edition. 1942.

PHOTOGRAPHY.

BAKER, T. THORNE. Photographic Emulsion Technique. 1941.

GREENLEAF, ALLEN R. Chemistry for Photographers. 1941.

PHYSICS.

De Ment, Jack. Fluorescent Chemicals. 1942.
Gage, Simon Henry. The Microscope. Seventeenth Edition, Revised. 1941.
Pearse, R. W. B., and A. G. Graydon. Identification of Molecular Spectra. 1941.
Richtmyer, F. K., and E. H. Kennard. Introduction to Modern Physics. 1942.
Watson, F. R. Acoustics of Buildings. Third Edition. 1941.

TECHNOLOGICAL HISTORY.

Rogers, Agnes. From Man to Machine. 1941.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Development of Resistance in Staphylococci to Natural Inhibitory Substances.—R. Lloyd Phillips (with the technical assistance of Lucy H. Barnes). *Staphylococci* may be resistant to complete inhibition by gramicidin. This was observed in this laboratory when a series of tests was made to determine the germicidal properties of this material by the F. D. A. phenol coefficient method (McCulloch, "Disinfection and Sterilization," Lea and Febiger, 1936). A similar observation has been made by Tishler, Stokes, Trenner and Conn (*Journal of Biological Chemistry*, **141**: 197, 1941).

In determining the phenol coefficient it was found that a loop transfer of the *Staphylococci* in the gramicidin solution often carried over enough gramicidin to inhibit growth in the transfer tubes for 48 hours although growth appeared in these same tubes after four to five days. The ability of the organisms to initiate growth after such long delay prompted further investigation.

It was found that transfer of *Staphylococci* in increasing amount of gramicidin resulted in the production of strains very resistant to the inhibiting action of gramicidin.

Experimental.—The material used in this study was prepared from the active fraction of Hoogerheide (*Journal of Bacteriology*, **40**: 415, 1940) (Journal of the Franklin Institute, **229**: 677, May 1940), and has proved to be identical with Dubos' gramicidin (Hotchkiss and Dubos, *Journal of Biological Chemistry*, **141**: 155, 1941) (Tishler, Stokes, Trenner and Conn, cited above).

The experiments were carried out in tubes containing 10 ml. of 0.1 per cent dextrose, beef extract broth, without added salt, at pH 7.2. The inoculum used was always 0.1 ml. of a 24 hour broth culture. The gramicidin stock used was 2 per cent. in 70 per cent. alcohol and appropriate dilutions were made in sterile distilled water. The distilled water suspension of gramicidin was added aseptically to the tubes of medium immediately before inoculation in quantities necessary to give the desired concentration of gramicidin in the transfer tubes. Usually each transfer was made in a series of tubes containing different amounts of gramicidin. The tube containing the highest amount of gramicidin which allowed growth in 24 hours was used as the source of inoculum for the next transfer in still higher concentrations of gramicidin. The original culture was carried simultaneously through the same number of transfers and titred at each transfer through the range of gramicidin concentrations giving inhibition for 48 hours.

Of the ten strains of *Staphylococci*,[1] strain 7893 was obtained from Dr. George H. Chapman, Clinical Research Laboratories, New York; strains 4776, 6339, and 6343 were obtained from the American Type Culture Collection; strains 93, 133, 40, 66, and 61 were obtained from the Department of Bacteriology, University of Delaware, and had originally been isolated from bovine mastitis; strain 4 was isolated in this laboratory from a human arm abscess.

As a result of the treatment each of these cultures became resistant to amounts of gramicidin from 660 to 1000 micrograms per ml. Broth containing such amounts of gramicidin is itself quite opaque and growth cannot be read directly. However, plate counts showed that resistant cultures remained alive in these amounts of gramicidin for as long as 21 days with numbers of viable organisms equal to those of control cultures in normal broth.

The amounts of gramicidin required to inhibit the original cultures and amounts failing to inhibit the resistant cultures are given in Table I. The cultures acquired resistance to 1000 micrograms per ml. of gramicidin and treatment was arbitrarily stopped at that point.

The resistant cultures did not vary from the original cultures in their abilities to ferment dextrose, levulose, lactose, mannitol, mannose, trehalose, inulin, dextrin or sucrose with the exception of the No. 4 resistant culture which failed to ferment mannitol and lactose although the original culture fermented both.

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[1] We are indebted to Dr. George H. Chapman, of the Clinical Research Laboratories, New York, N. Y., and to Dr. J. R. Hay and Dr. C. C. Palmer of the University of Delaware for furnishing us with the cultures indicated.

All of the cultures, original and resistant, produced coagulase with the exception of No. 6343 resistant culture which had lost this characteristic. All the cultures produced hemolysis on rabbit blood agar plates and grew on Bromthymol-blue-lactose-agar plates.

Table I.

Strain	Gramicidin Required to Inhibit Original Culture for 48 Hours.	Gramicidin Failing to Inhibit Resistant Cultures.
7893	10 micrograms per ml.	660 micrograms per ml.
4776	20 " " "	1000 " " "
6339	8 " " "	1000 " " "
6343	6 " " "	1000 " " "
93	12 " " "	1000 " " "
133	12 " " "	1000 " " "
40	5 " " "	1000 " " "
66	12 " " "	1000 " " "
61	8 " " "	1000 " " "
4	5 " " "	1000 " " "

On crystal-violet agar, the reaction of resistant and non-resistant cultures was the same with the exception of No. 4 resistant culture which produced a paler color than the parent strain.

During the development of the resistant cultures, frequent colonial variants appeared. Because preliminary tests had shown that variation occurred in resistant cultures, before beginning transfer in gramicidin, all cultures were plated 10 successive times and smooth colonies picked to insure the use of a smooth strain in each case. Plates were made of the transfers in gramicidin and of the original cultures for comparison. Many sets of transfers were held for as long as three weeks and replated at intervals during that time. Ageing for this period produced no variants or no variants more stable than those encountered in earlier plates. The colony types encountered have been described by Biggers, Boland and O'Meara (*Journal of Pathology and Bacteriology*, **30**: 261, 1927), Pinner and Voldrich (*Journal of Infectious Diseases*, **50**: 185, 1932), and Hoffstadt and Youmans (*Journal* *of Infectious Diseases*, **51**: 216, 1932) and need not be described in detail here. The variants encountered most frequently are:

7893—A typical smooth *aureus* strain which showed no tendency to variation in the original culture. Resistant cultures showed variants which were white, white with orange sectors, white crenated colonies and orange crenated colonies.

4776—An *aureus* strain. In resistant cultures grown in 50 to 400 micrograms gramicidin per ml., as high as 90 per cent. of the colonies growing on agar plates were rough, crenated with a few white colonies. The culture resistant to 1000 micrograms per ml. gramicidin was smooth *aureus*, indistinguishable from the original strain.

6339—An *aureus* strain. During daily transfer this strain produced little color. The resistant culture appeared as a mixture of colored and white smooth colonies.

6343—This *aureus* strain was obtained dried and produced many viscid colonies upon subculture. These disappeared during daily transfer and by picking normal colonies for transfer. The resistant cultures showed numerous crenated variants. In the higher concentrations of gramicidin, the culture was smooth with only a slight tendency to rough appearance; but most of the colonies were viscid and lifted off the medium whole.

93—A cream colored strain. During the tests, the original strain threw off occasional yellow colonies. Yellow colonies in the resistant cultures were more frequent and the most resistant strain was completely lemon yellow.

133—A cream colored strain. In gramicidin, cream and *aureus* colonies were produced with occasional lightly crenated colonies of both types.

40—A cream colored strain. In the presence of gramicidin, this strain gradually produced more color until in the presence of 300 micrograms per ml., it was a fully developed *aureus* strain producing color in 24 hours. The original remained cream colored with an occasional colony showing a slightly deeper colored center.

66—A smooth cream-white strain. Viscid variants appeared in 600 micrograms per ml. gramicidin. The culture remained cream-white.

61—This culture had a tendency to produce color as shown by the frequent appearance of colonies with a light tan center. In gramicidin-resistant cultures, fully developed *aureus* colonies soon appeared along with cream smooth and cream crenated colonies. The most resistant culture was entirely smooth *aureus*.

4—An *aureus* strain. In gramicidin, a few white colonies appeared at various times. As many as 70 per cent. white colonies appeared in the lower dilutions of gramicidin.

The strongest tendency toward variation appeared when the cultures were grown in concentrations of gramicidin between 100 and 400 micrograms per ml. In the presence of higher concentrations these strains were more stable with the appearance of either fewer rough or markedly smoother colonies.

The colonies from gramicidin-treated transfers were as a rule smaller and grew more slowly than the originals. The most resistant culture was indistinguishable from the original except that microscopic examination showed a preponderance of coarsely granular colonies as contrasted to the finely granular colonies of the original strain.

The appearance of granular colonies occurred with sufficient regularity to indicate the possibility that this accompanied the development of resistance and that organisms from these colonies might perhaps be responsible for the resistance to gramicidin. However, organisms from both finely granular and coarsely granular colonies picked from resistant cultures were resistant to inhibition by gramicidin.

Attempts were made to maintain rough strains by picking rough colonies into gramicidin broth, but such broth cultures always gave smooth colonies even at the first transfer and rough colonies grown in increasing amounts of gramicidin finally gave rise to completely smooth strains. White colonies picked from smooth strains were found to be more stable than pigmented colonies.

The changes may be summarized as follows: (1) the development of relatively stable white colonies from an *aureus* culture; (2) the intensification of a tendency to produce color; (3) the production of non-stable rough variants as well as crenated and viscid variants; (4) the appearance of coarsely granular colonies.

The gramicidin-resistant cultures were also resistant to tyrothricin, the active material from which gramicidin and tyrocidin are extracted (Dubos and Hotchkiss, *Journal of Experimental Medicine*, **73**: 629, 1941). Strains of *Staphylococci* became resistant to inhibition by tyrothricin also when they were transferred in media containing increasing amounts of this material.

Summary.—Staphylococci grown in the presence of increasing amounts of gramicidin became resistant to inhibition by gramicidin.

The development of resistance in successive transfers was accompanied by the production of colonial variants, especially in the lower concentrations of gramicidin.

Resistant cultures did not greatly differ from the original strains in the biochemical characteristics tested.

The State of Aggregation of Gelatin in Non-Gelating Systems.—Elmer O. Kraemer (*Journal of Physical Chemistry*, **46**: 177, 1942). In continuation of previous work on the viscosity, diffusional, and ultracentrifugal behavior of Eastman's de-ashed gelatin (Kraemer, *Journal of Physical Chemistry*, **45**: 660, 1941) in non-gelating solvents, further results are presented for gelatin in one molar thiourea, two molar potassium thiocyanate, and other solvents. For the non-gelating solutions, the temperature coefficient of viscosity, the sedimentation constant, and the diffusion constant all indicate that gelatin under these conditions is molecularly dispersed, as appears to be the case above the gelation temperature in gelating solutions. Minor differences in these physical properties for the various solutions are probably due to differences in solvation or degree of coiling of the gelatin molecules, or to combination of the gelatin with the gelation-inhibiting agents. The molecular weight of the Eastman gelatin is about 110,000, and the frictional ratio is about 4, which is an unusually high value for a protein.

Studies on the Hemolytic Streptococcus. V. The Electrophoretic Isolation of the Erythrogenic Toxin of Scarlet Fever and the Determination of its Chemical and Physical Properties.—L. E. KREJCI, A. H. STOCK,[2] E. B. SANIGAR AND E. O. KRAEMER (*Journal of Biological Chemistry*, **142**: 785, 1942). Scarlet fever (erythrogenic) toxin of a high degree of activity was isolated chemically from filtrates of cultures of the NY5 strain of hemolytic *Streptococcus* grown in the diffusate of tryptic digest broth. The toxin prepared in this way was analyzed by electrophoresis in the Tiselius apparatus and was found to contain five electrophoretically distinct constituents, all migrating toward the anode at pH 7. Four, designated I, II, III, and IV, were colorless; the fifth, a brownish pigment, was present in too low a concentration to influence the refractive index diagrams. A sixth (colorless) constituent, designated N, was present in one of four preparations.

The slowest migrating constituent (IV) was identified by means of skin tests as the erythrogenic toxic constituent. It was isolated by electrophoresis and found to possess toxic activity of between 100 and 150 million skin test doses per milligram. Chemical analyses, the property of heat coagulability, and the ultraviolet absorption spectrum show that the toxic constituent is a protein. Chemical tests indicate that constituent II is an acid-precipitable nucleoprotein-like substance, and that constituent I may be a polysaccharide.

The mobility of the electrophoretically separated toxin (constituent IV) is -1.93×10^{-5} cm.^{2} volt^{-1} sec.$^{-1}$ at pH 7 in the buffer solution used. The mobilities of the other four colorless constituents under the same conditions are as follows: constituent N: -3×10^{-5}; constituent III: -5.2×10^{-5}; constituent II: -6.8×10^{-5}; and constituent I: -10×10^{-5}.

The sedimentation constant of the electrophoretically purified toxin is 2.7×10^{-13} Svedberg units; the diffusion constant, calculated from the sedimentation diagrams, is 9.5×10^{-7} cm.^{2} sec.$^{-1}$. Assuming a partial specific volume of .736, the value for diphtheria toxin, these figures indicate a molecular weight of 27,000 and a frictional ratio of 1.1. The sedimentation diagrams give evidence of a high degree of homogeneity.

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[2] Of the Department of Pathology, Children's Hospital and University of Pittsburgh, Pittsburgh, Pennsylvania, where the preparation of the toxin, the skin tests, and most of the chemical analyses were carried out.

The high activity of the electrophoretically isolated toxin and its substantial homogeneity with respect to both electrophoresis and sedimentation are evidence that this material is probably pure scarlet fever (erythrogenic) toxin.

BOOK REVIEWS.

ENGINEERING SURVEYS, by Harry Rubey, George Edward Lommel and Marion Wesley Todd. Elementary and Applied, 664 pages, illustrations, tables and index—170 pages. 13 × 20 cms. New York, The MacMillan Company, 1942. Price $4.50.

Application of a knowledge of engineering surveying provides probably more diversified benefits than any other one subject. Such knowledge has direct bearing on different kinds of engineering including civil, mining, agricultural, electrical, mechanical and chemical. Many phases of surveying are advancing so rapidly that they must be followed in technical periodicals and in engineering practice rather than in textbooks. To keep abreast of this movement requires a sound training in fundamentals. The book at hand endeavors to give such a treatment. It covers a combined course of elementary and applied or higher surveying. As to its aims, the following is taken from the Preface, "The present volume aims to present the entire subject as it is taught throughout the United States, and as specified in—the minimum course content adopted by the Second National Surveying Teachers' Conference of the Society for the Promotion of Engineering Education."

Dealing with plane surveys the book follows a logical progression from the simple to the more complex. The methods employed, the use of the various instruments, the difficulties encountered in surveying, the different types of surveys and the technique employed in each are all covered. The book is replete with illustrations of various kinds, problems for exercise, and lists of references. Theories are closely interwoven with practice so that the book is essentially instructive in a practical sense. The subject index in the back adds greatly in making the book a valuable text and reference work.

R. H. OPPERMANN.

COLLEGE PHYSICS, by A. Wilmer Duff and Morton Masius. 588 pages, illustrations, 15 × 23 cms. New York, London, Toronto; Longmans, Green and Co., 1941. Price $3.80.

The college physics text has always been of extreme importance in education. It is the basic tool for a scientific education. It must therefore have many qualifications, some of which may be rated differently according to the viewpoint of the user, be he teacher or student. But college physics texts have one thing in common with other texts. A good one lasts and is used for a comparatively long time. It is worth while to revise it so that it is up-to-date and will reflect the latest in teaching experience. The book at hand is a revision of a book by Dr. Duff, one of the present authors.

The plan of presentation embodied herein is founded on a classification of the kinds of energy, plus historical order of concepts and the general usefulness of the concepts in further development of the subject. Thus, mechanics is the first topic, mechanical energy having all of these qualities. As covered here it is divided into the study of statics, dynamics, and mechanics of fluids including wave motion. Sound energy is next taken up and in the very first part of this, there is illustrated what is typical throughout the book. In very plain language it is explained that what is meant by sound energy is objective sound, or the external source of the hearing sensation. Following this, heat energy is taken up, and then electricity and magnetism, and finally flight.

The book deals with fundamentals only. The subject matter is clearly and simply presented. Carefully constructed exercises appear at the end of each chapter and these are supplemented by additional exercises appearing at the end of the book. There is also to be found here a subject index, adding to the value of the book for reference work. Students, teachers and others interested in the subject of fundamental physics will find this book worth their consideration.

R. H. OPPERMANN.

MODERN PULP AND PAPER MAKING, a practical treatise, by G. S. Witham, Sr. Second edition. 705 pages, illustrations and plates, 16 × 24 cms. New York, Reinhold Publishing Corp., 1942. Price $6.75.

Paper has assumed an importance in our life, unsurpassed by many other commodities. It is so common, its many varieties are so readily adaptable, that it falls into the category of being taken for granted. Many of us do not realize what there is in back of a sheet of paper, the ingenuity, industry, and resourcefulness. Even those engaged in branches of the paper industry often are not familiar with its complexity, as for instance, conditions and situations in one location can be entirely different than in another.

The first edition of this book was written in 1920. It had in its title, "A Practical Treatise," as this second edition has, and in accordance the contents are directly applicable to the paper mill. The chemistry and physics involved in pulp and paper making are only treated on here in-so-far as is necessary to bring out the practical aspects. The book is entirely one having to do with the actual accomplishment of manufacture.

The first part is devoted to rather brief descriptive matter of the processes by which pulp is produced, the materials of pulp, and the varieties of paper. From here on the span of manufacture is treated. General observations are given with regard to the sawmill in the various paper-making regions and there is described the delivery to and the general operations in the wood room, after which is discussed the processes and equipment required for the manufacture of chemical pulp by the sulphite process. A separate chapter is devoted to the acid plant where the acid liquid used in the sulphite digesters is prepared. The alkaline processes—soda and sulphate or kraft—are considered together. Groundwood or mechanical pulp is next taken up and this section gives very interesting accounts of different types of grinders. This is followed by explanations of screening and refining, and bleaching of pulp which ends the sections of the book on the various processes for making pulp.

The manufacture of paper is then discussed, beginning with the beater room, and proceeding through the machine room, the finishing room and converting and coating. Supplementing this there is a very interesting chapter on paper defects, their cause and cure. The latter part of the book has to do with the paper manufacturing establishment itself, its general design, the operating force, the power plant in which certain aspects are noted which are special to the paper plant, and the testing of paper and paper materials including control of the mill.

The book has many illustrations, both drawings and photographs, which together with good clarity in presenting the subject, make it outstanding in value to the paper mill worker, the engineer, the superintendent—in fact everyone whose interest lies wholly or partially with the paper industry.

R. H. OPPERMANN.

EXPERIMENTAL PHYSICAL CHEMISTRY, by W. G. Palmer. 321 pages, illustrations, 15 × 23 cms. Cambridge, The University Press, 1941. Price $2.75.

The supplementing of classroom instruction with the performance of prescribed experiments is an absolute necessity in the study of Physical Chemistry. Because of this, great care must be exercised in arranging every detail of the laboratory work, particularly with regard to a knowledge of the apparatus, the method of the experiments and the significance of each, and the sequence of the work. Any book on the subject must therefore be of a supplemental nature to the main theoretical treatment.

The book at hand was designed primarily to fit in with British courses in Universities and Colleges. The author states that while much of the work has been found suitable for beginners of the subject, who are candidates for an Honours Degree through the Natural Sciences Tripos, a number of experiments are described which are appropriate to more advanced students. This is not to indicate, however, that the book should be precluded from use elsewhere than in Britain. A hasty perusal of its contents reveals possibilities which are verified by closer examination.

There are eight chapters in the book. It begins with illustrations of ways of making for gases and vapours the two estimations of weight and volume, to which all determinations of density must reduce. Crystallization and the properties of crystals is the next topic. In covering solutions and solubility, the subject is divided into three parts—simple systems in which one solution is in equilibrium with one or more pure phases (e.g., gases and water, salts and water, at normal temperatures); mutual systems where two or more saturated solutions are in mutual equilibrium and pure phases may be absent (e.g., partially miscible liquids, and equilibria between liquid and solid solutions); dilute solutions. Subsequent chapters deal with thermochemistry; ionization divided into the conductance of electrolytic solutions, the creation of electromotive force, and hydrogen indicators and buffer solutions; velocity of chemical reaction; surface chemistry including surface free energies, interfacial tension, adsorption at the solid-liquid interface, the preparation and properties of suspensoid colloids and heterogeneous or contact catalysis.

There is considerable theoretical treatment before the explanation of experiments, which however requires a good chemical background for thorough understanding and use in the experiments. The experimental procedure is in the main simple, containing in some places a free or liberal style of explanation and direction. The author has followed the practice of working out examples to make lasting impressions.

The book is worthy of examination for determining its fitness for use in courses on the subject. It also may offer suggestions which may be valuable.

R. H. OPPERMANN.

PRINCIPLES OF MECHANICS, by J. L. Synge and B. A. Griffith, first edition. 514 pages, illustrations, 16 × 24 cms. New York and London, McGraw-Hill Book Company, Inc., 1942. Price $4.50.

It is always fascinating to examine the structure of a text on a subject like mechanics for there is revealed not only the divisions and arrangement of the material for appraisal as to the effectiveness on the student, but the plan and foresight of the author. With regard to the latter, the structure of this book reveals a multitude of factors that must have been considered before the writing was undertaken.

The first division of the book is devoted to Plane Mechanics. The treatment rises from a recording of some broad philosophical ideas leading to more specific foundations of mechanics. This lays the foundations of a logical structure. The plane mechanics is then taken up in earnest and the convenience of learning the subject first through this division is apparent. The mathematics are comparatively simple. Methods of an applications in plane statics are taken up and preparation is made for the study of dynamics by developing some results in plane kinematus. Then, similar to the study of statics, the methods of an applications in plane dynamics are taken up. This, however, is followed by the topic "plane impulsive motion" where the concept of impulsive force is recognized and there is repeated for such forces results obtained for ordinary forces. This concludes the development of mechanics in two dimensions.

233
42

In preparation for the treatment of mechanics in space (part 2) a review is made of the vector rotation, insofar as the products of vectors are concerned. It is noted that a knowledge of three-dimensional geometry is also required for this study. After considering statics in space, the approach to the study of dynamics in space is through kinematics which provides a simple way of describing the motions of particles and of rigid bodies, and methods of calculating kinetic energy and angular momentum. Applications in dynamics in spcae include a very interesting treatment of the motion of a charged particle in an electromagnetic field. The last two chapters of part 2 and of the book are on Lagrange's equations and the special theory of relativity in which gravitational attraction plays no part.

The chapters proceed in logical and even progression. Many of them have at the beginning an introductory note which is generally descriptive. Those not having such note begin with elementary theoretical treatment. Great care is exhibited in the approach to new topics. A novel and commendable feature of each chapter is the summary of the ground covered in a sort of abstract or notebook construction, easy to remember. This is found at the end of the chapters where also can be found numerous exercises. The two parts or divisions of the book are independent of each other which may assist its fitness to certain courses. Throughout the book there is revealed a keen insight to the needs and situation of the student and great effort is made to present the subject in a satisfactory manner.

R. H. OPPERMANN.

PUBLICATIONS RECEIVED.

Principles of Mechanics, by J. L. Synge and B. A. Griffith. First edition, 514 pages, illustrations. 16 × 24 cms. New York and London, McGraw-Hill Book Company, Inc., 1942. Price $4.50.

The Condensed Chemical Dictionary, edited by Thomas C. Gregory of the Chemical Engineering Catalog. Third edition, 756 pages, 16 × 23 cms. New York, Reinhold Publishing Corporation, 1942. Price $12.00.

Bell Telephone System, Monographs: B-1319, Contribution of Statistics to the Science of Engineering, by Walter A. Shewhart. 28 pages, illustrations. B-1320, Theory of Antennas of Arbitrary Size and Shape, by S. A. Schelkunoff. 76 pages, illustrations. B-1321, The Thermal Expansion of Pure Metals: Copper, Gold, Aluminum, Nickel, and Iron, by F. C. Nix and D. MacNair. 20 pages, illustrations. B-1322, A Visual Test for Calcium in Lead, by Earle E. Schumacher and G. M. Bouton. 5 pages, illustrations. B-1323, The Reliability of Holding Time Measurements, by Roger I. Wilkinson. 40 pages, illustrations. B-1324, Television—The Scanning Process, by Pierre Mertz. 24 pages, illustrations. Six pamphlets, 15 × 23 cms. New York, Bell Laboratories, 1941.

College Physics, by William T. McNiff. 657 pages, illustrations, 16 × 24 cms. New York, Fordham University Press, 1942. Price $4.00.

Scientists Face the World of 1942, essays by Karl T. Compton, Robert W. Trullinger, and Vannevar Bush. 80 pages, 16 × 24 cms. New Brunswick, Rutgers University Press, 1942. Price $1.25.

Manufacture of Soda, With Special Reference to the Ammonia Process, A Practical Treatise, by Te-Pang Hou, Ph.D. Second edition. 590 pages, illustrations and tables, 16 × 24 cms. New York, Reinhold Publishing Corporation, 1942. Price $9.50.

CURRENT TOPICS.

Mercury Poisoning as a Mining Hazard.—This is the title of a Bureau of Mines information circular by Sara J. Davenport and D. Harrington, which states that an increase in the occurrence of mercury poisoning among workers in mines and allied industries may be expected as the result of growing demands for this strategic metal in war time industries. Mercury vaporizes readily, and small concentrations of its fumes or dust may cause serious physical disturbances which sometimes affect the brain and nervous system of workers according to the Bureau. In mercury mining the most effective way to control and reduce poisoning of human beings is by thorough and adequate ventilation of mines which removes fumes and dust and cools the air. For preparing and purifying the metal, the Bureau recommends the use of airtight furnaces, condensers and retorts, with enough negative pressure to prevent the escape of dust or fumes. In other occupations where mercury is handled, such as in the manufacture of explosives, prevention of mercury poisoning is a matter of "good housekeeping" and sanitation. Every care should be taken to prevent spilling of mercury droplets and to clean it up after it has been spilled. Floors, benches and other fixtures on which dust or droplets may accumulate should be cleaned daily. The main source of mercury is cinnebar, and poisoning may develop in mining from handling ore that contains free mercury, from dust and fumes that escape during retorting the ore and condensing the vapor, and from the handling of liquid mercury. In addition to workers in mines and processing plants, other persons are possible victims of mercurial poisoning including physicists, chemists, physicians, dentists and natural science teachers. Mercury enters the body chiefly by inhalation of dust and vapor and by swallowing as with food, although dust can enter through the skin. Characteristic symptoms are inflammation of the mouth, psychic irritability and muscular tremor with the subsequent varying effects.

R. H. O.

Cylinder Columns Carry Shipyard Cranes.—(*Engineering News-Record*, Vol. 128, No. 9.) Without precedent as a guide, unbraced column supports 93 feet high have been used for overhead cranes at a shipyard where clearances were limited by reason of the desire to utilize piling already in place. The columns are cylindrical in shape with bases firmly bolted to a concrete slab capping a cluster of wooden piles. The structure supporting each crane is thus a row of vertical cantilevers tied together at the top by a continuous steel girder 630 feet in length supported at six points 110 feet apart and having a 40 foot overhang at each end. At this yard there are to be three shipways; two are already built and a third is under construction. Columns supporting the overhead crane are the same on outer and inner rows but the continuous girders are doubled on those rows of columns that support two crane rails. In other words, each crane rail has its own supporting girder. Where two girders are put on the same row of columns, the webs are interconnected by bracing to gain the added stiffness of what is virtually a box girder. All parts of this steel structure are welded. Prior to erection of the girders, a test load was applied at the top of one of the columns to observe its action under bending stresses. The load was applied by a cable running diagonally to the top, pulling in a plane at right angles to the girder center line. Observations were made under supervision of Prof. R. R. Martel, California Institute of Technology, on test loads with a horizontal component of 35,000 lb. This produced a $3\frac{5}{8}$ in. deflection at the top of the 93 ft. column. The column itself is made of plates rolled to a prescribed diameter of 95 in. The test results indicated very nominal stresses for a combination of dead and live loads with a horizontal thrust of 0.2 g. The calculated vibrational period of the completed 630 foot structure is about 1 sec. with the crane spanning the space between the two rails. Movements in the girders resultant from wind or such shock or impact as there is in crane operation are only fractions of an inch.

R. H. O.

American Manganese.—A domestic manganese war program of sufficient scope to free American steel and alloy production from its present reliance on foreign sources has been developed by the United States Bureau of Mines. The program, designed to provide sufficient manganese to produce 87 million tons of steel annually, covers utilization of low-grade domestic manganese. The Bureau is to make available its knowledge, experience, technical personnel, and processes to industry so that industry can benefit from the work the Bureau has done, and to supervise operations if industry so requests. Only in the event industry is unable to undertake the production rapidly, the Bureau is prepared to assume responsibility for the necessary production itself. The engineers of the Bureau report that they do not hesitate to stake their professional reputations on the feasibility and workability of the processes they have tested.

R. H. O.

Hydrogen for High-Frequency Cooling.—(*Power*, Vol. 86, No. 2.) High-frequency current for metallurgical applications, heat treating, melting, and heating finds ever increasing use. Power of 1000 kw. at 10,000 cycles is now possible, and for power above 100 kw. and frequencies up to approximately 10,000 cycles, the inductor type of electromagnetic generator is universally used. One limitation in the size of these generators is the high windage loss which limits the rotor diameter. With hydrogen cooling these high windage losses are reduced to a fraction of their value, and the efficiency of the machine is considerably improved. Another noticeable result obtained by the use of hydrogen cooling is the marked decrease in the noise, a pronounced characteristic of high-speed generators. Furthermore, sealed enclosures prevent dirt from reaching the windings and the machine remains permanently clean. This feature is particularly valuable in metallurgical plants where air filters are often found necessary in rotating electric machines. A 500 kw., 9600 cycle hydrogen-cooled generator has been in service for more than one year in the plant of Budd Induction Heating, Inc., in Detroit. A similar machine has recently been installed at the Caterpillar Tractor Company, Peoria, Illinois. In a machine for generating 10,000 cycles, the poles of the rotor and the air gap must be very small. This means highly accurate machining and alignment in order to reduce the unbalanced magnetic pull to a minimum. With the small air gap for a small displacement of the rotor due to the bearing clearance, this unbalanced magnetic pull is sufficient to overcome the weight of the rotor. The bearings are designed to carry load in any radial direction and can carry a much higher load than conventional bearings. The stater core is built of thin laminations. Because of high-frequency current, large losses are confined to a small volume of material. Water cooling is the most effective means of dissipating stator copper and iron losses.

R. H. O.

Sag by Telephone.—(*Electric Light and Power*, Vol. 20, No. 2.) Engineers and linemen have long known that "a straight line is not the safest distance between two points" and have taken heed when sagging conductors on transmission lines. In recent years more emphasis has been placed on the proper sagging of distribution conductors and a closer relationship between service reliability and conductor sagging has been recognized. The usual method of sagging a long section of line is to select one "ruling span." Horizontal sagging sticks are placed on the poles on either side of this span the sticks representing the point to which conductors should be sagged. Sighting over these sticks, a lineman signals to men stationed on the ground along the line and these men relay signals to the pulling crew. To facilitate this signaling process, Line Foreman Albert Stress of Watertown, N. Y. (Niagra Hudson System), devised an ingenious method of controlling sagging operations by telephone. Battery operated phones are connected to the wire being sagged and by means of these Foreman Stress talks to the man sighting on the pole at the ruling span. Standing at the pulling end, he coördinates sighting and pulling from one point.

R. H. O.

Variable-Speed Drive for United States Army Air Corps Wind Tunnel at Wright Field, Dayton, Ohio.—A. D. DICKEY, C. M. LAFOON, L. A. KILGORE. (*Electrical Engineering—Transactions of A. I. E. E.*—Vol. 61, No. 3.) The new wind tunnel of the United States Army Air Corps at Dayton, Ohio, which is scheduled for operation shortly, is an example of a design and construction problem in connection with our national defense program that machinery manufacturers are required to solve. Wind velocities of 400 miles per hour and greater are to be obtained for testing large size airplane parts and models. The air is to be circulated by two propeller assemblies mounted on a common shaft and operated in series to produce the necessary high pressure. The propeller assembly is driven by a 40,000 horsepower, variable speed, wound rotor type induction motor, which has a top rotational speed of 300 r.p.m. (327 r.p.m. synchronous speed) and is the largest unit of this type built to date. In order to maintain speed practically constant at any value over a wide speed range with minimum energy requirements, a speed control system was chosen which provided for returning the power from the rotor of the main induction motor to the a.-c. supply system. The main rotating auxiliary equipment required to accomplish this result consists of two motor generator sets. The a.-c. elements of both sets are salient-pole synchronous units with d.-c. excitation. In one set, the a.-c. motor receives its power supply from the rotor of the main drive motor, and operates as a synchronous machine at the slip frequency over the entire speed range of the main motor. The second set has the a.-c. generator electrically connected to the 60-cycle main supply system, and operates at constant synchronous speed. Through the interconnecting of these machines, control of the speed is by the field of the d.-c. motors.

R. H. O.